Weighted Kripke Structures and Refinement of Models

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We consider binary relations weighted over frames to express intermediate degree of relationship. We reformulate the notions of transition systems, Kripke structures and simulations between them to formulate multi-valued model checking of modal μ-calculus. Discussion on the use of multi-valued model checking to refine models in the usual sense follows.

1 Introduction

Intermediate truth values are favored. In many situations the notions of functions, relations etc. are re-invented upon intermediate or multiple truth values. Those situations are named under the adjectives “multi-valued,” “fuzzy,” “probabilistic,” and “quantum.” The meaning of multi-valuedness varies in each context, but there seems to be a common basis for all these situations which is based upon multi-valued relation.

In fact, a number of ways in formulating such notions have been proposed and studied independently. We seek for a better, more general formulation.

We introduce binary relations weighted over complete Heyting algebras to express intermediate degree of relationship. We extend the notion of Kripke structures based on weighted binary relations, and and that of simulations between them to formulate multi-valued model checking of modal μ-calculus[4]. We also give the semantics for modal μ-calculus and that for linear-time modal μ-calculus over the extended Kripke structures. Our main theorem here is the simulation theorem in this context. We also mention an application to model refinements [1, 8].

2 Weighted Relations

In this section, we review the category $\mathbf{Mat}(L)$, in which our work in this paper is formulated, following Johnstone[3], §A3.2, A3.3. This is a category (and even an allegory) whose objects are small sets and morphisms are binary relations weighted over the *frame* (or complete Heyting algebra) L. In the sequel, references like §A3.2 etc., are all references to paragraphs in [3].

Relations from a set X to Y are usually formulated by a subset of $X \times Y$, which could be considered as a function R: $X \times Y \longrightarrow 2$, where 2 is the set with exactly two elements $\top$ and $\bot$. For each $x \in X$ and $y \in Y$, R holds between them ($R(x, y) = \top$) or does not ($R(x, y) = \bot$). To consider intermediate status, 2 could be replaced by a frame L to obtain relations weighted over L. In fact, small sets and relations weighted over L form an *allegory*[2, 3] $\mathbf{Mat}(L)$.

Definition 1 (Allegory $\mathbf{Mat}(L)$)**.** *Let L be a frame. The following data form an allegory $\mathbf{Mat}(L)$[3].*

- $\mathrm{ob}(\mathbf{Mat}(L)) \stackrel{\mathrm{def}}{=}$ *small sets.*
- $\mathbf{Mat}(L)(X, Y) \stackrel{\mathrm{def}}{=} \{ f \colon X \times Y \longrightarrow L \mid f$ *is a function*$\}$, *for* X, $Y \in \mathrm{ob}(\mathbf{Mat}(L))$.
- $\mathrm{id}_X(x, x') \stackrel{\mathrm{def}}{=} \top_L$, *if* $x = x'$, $\bot_L$, *otherwise.*

- $(S \circ R)(x,z) \stackrel{\text{def}}{=} \bigvee_L \{ S(y,z) \wedge_L R(x,y) \mid y \in Y \}$, *for* $R \in \mathbf{Mat}(L)(X,Y)$ *and* $S \in \mathbf{Mat}(L)(Y,Z)$.
- $R^\circ(y,x) \stackrel{\text{def}}{=} R(x,y)$.
- $(R \wedge S)(x,y) \stackrel{\text{def}}{=} R(x,y) \wedge_L S(x,y)$.

The arrow of $\mathbf{Mat}(L)$ is called *L-weighted relation (L-relation).* We write $R\colon X \nrightarrow Y$ for $R \in \mathbf{Mat}(L)(X,Y)$, when L is clearly determined from the context. We sometimes regard $R \in [S,L]$ as an L-relation $R\colon 1 \longrightarrow S$ where 1 is the set with one element.

At a first glance one might naïvely conclude it is sufficient for L to be a partially ordered set, but for the well-formedness of the composition, it has to be a complete lattice and for the associativity of composition, it has to be a frame.

The rest of this section is devoted to show why we look at the allegory $\mathbf{Mat}(L)$. Now, the category **Rel** of small sets and binary relations has a special connection to **Set** as follows. **Rel** is an allegory which has a unit and satisfies the property "tabularity," and **Set** is a category which satisfies the property called "regularity." There is an operation Map on a tabular allegory with unit to form a regular category, and an Rel on a regular category to form an allegory with a unit. These satisfy $\mathbf{Rel} \cong \mathrm{Rel}(\mathbf{Set})$ and $\mathbf{Set} \cong \mathrm{Map}(\mathbf{Rel})$.

Now it would be natural to seek for a regular category which is what $\mathbf{Mat}(L)$ is to as **Rel** is to **Set**. As we explain below, $\mathbf{Mat}(L)$ itself is not tabular, but its Cauchy completion $\mathbf{Mat}(L)[\check{\mathcal{S}}]$ with respect to the set of all symmetric idempotents is and it also has a unit. As the name suggests, $\mathbf{Mat}(L)$ can be embedded into $\mathbf{Mat}(L)[\check{\mathcal{S}}]$, so the result of applying Map operation to it deserves to be called a category of sets and "functions" weighted over L.

A *map* in an allegory is a morphism in it which has a right adjoint. A right adjoint of a morphism φ in an allegory must be φ° (§A3.2.3), so φ is a map if and only if $\mathrm{id}_A \leq \varphi^\circ\varphi$ and $\varphi\varphi^\circ \leq \mathrm{id}_B$. A leading example of allegories is of course **Rel** of small sets and binary relations. Maps in **Rel** are exactly functions, so if one extracts maps from **Rel**, one would obtain a category equivalent to **Set**. Such an operation Map to allegories is important.

There is also an operation Rel on "regular" categories (§A1.3) for which $\mathrm{Rel}(\mathbf{Set}) = \mathbf{Rel}$. This is based on tables of binary relation. A table of a binary relation R from A to B is the subset $\{ (a,b) \mid a \; R \; b \}$ of $A \times B$. Naturally attached to the table is the restrictions $(a,b) \mapsto a$ and $(a,b) \mapsto b$ of projections to the table. Generalizing this to arbitrary allegories, a *tabulation* of a morphism $\varphi\colon A \nrightarrow B$ in an allegory is a triple (C,f,g) of an object C and maps $f\colon C \longrightarrow A$ and $g\colon C \longrightarrow B$ subject to $\phi = g \circ f^\circ$ and $(f^\circ \circ f) \wedge (g^\circ \circ g) = \mathrm{id}_C$. An allegory is *tabular* if all its morphisms has a tabulation.

It is known that $\mathbf{Mat}(L)$ is not tabular (§A.3.2), but its Cauchy completion (Karoubi envelope) $\mathbf{Mat}(L)[\check{\mathcal{S}}]$ with respect to the set $\mathcal{S}$ of all symmetric idempotents in $\mathbf{Mat}(L)$ is tabular. Moreover, the category of its maps $\mathrm{Map}(\mathbf{Mat}(L)[\check{\mathcal{S}}])$ is known to be a topos (§A.3.4). So this category is baptised as $\mathbf{Set}(L)$ and seems to be a natural basis for further development of our argument. For the argument in this paper, however, working within $\mathbf{Mat}(L)$ suffices. $\mathbf{Set}(L)$ is mentioned here only to say it is already known that the allegory $\mathbf{Mat}(L)$ where we work has a natural extension.

3 Weighted Kripke Structures

In this section, we introduce the notion of weighted Kripke structures based on weighted relations and simulations between them.

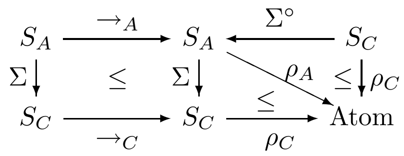
Definition 2 (L-Kripke structure)**.** *Let L be a frame. Let* Atom *be a signature (a set of atomic propositions). An L-weighted Kripke structure (L-Kripke structure) is a triple of a set S, an L-relation $\rightarrow$ upon S, and an L-relation $\rho\colon S \nrightarrow$* Atom*; elements of S are its states, $\rightarrow$ is its transition relation, and ρ is its labeling function.*

In a locally ordered category C in general, a simulation from an endomorphism $A\colon S_A \longrightarrow S_A$ to another endomorphism $C\colon S_C \longrightarrow S_C$ is defined to be a morphism $\Sigma\colon S_A \longrightarrow S_C$ such that $C \circ \Sigma \leq \Sigma \circ A$.

$\mathbf{Mat}(L)$ is an allegory, so it is a locally ordered category. Therefore, the above definition can be applied to $\mathbf{Mat}(L)$ to obtain a definition of simulations there. Adding the conditions for simulation of labeling functions (if a simulates c, the set of atomic propositions holding at c coincides the set of atomic propositions holding at a), we define the notion of simulations between L-Kripke structures as follows.

Definition 3 (Simulation)**.** *Let* $A = (S_A, \rightarrow_A, \rho_A)$ *and* $C = (S_C, \rightarrow_C, \rho_C)$ *be L-Kripke structures. An L-relation* $\Sigma\colon S_A \nrightarrow S_C$ *is defined to be a* simulation *from A to C if and only if the following holds.*

$$\begin{aligned} &\rightarrow_C \circ \Sigma \leq \Sigma \circ \rightarrow_A \\ &\rho_C \circ \Sigma \leq \rho_A \\ &\rho_A \circ \Sigma^\circ \leq \rho_C \end{aligned}$$



4 State Semantics

In this section, we define the truth values of formulae of modal μ-calculus over weighted Kripke structures, and formulate and prove the simulation theorem in this context.

Let L be a frame, $K = (S_K, \rightarrow_K, \rho_K)$ be an L-Kripke structure for a signature Atom, Var be the set of variables and $V\colon \mathrm{Var} \longrightarrow [S_K, L]$ be a valuation of variables, where $[S_K, L]$ is the frame of functions from S_K to L with the pointwise order. In this section, we shall define a weighted analogue of usual semantics for modal μ-calculus. This is called state semantics, while the semantics we introduce in the next section is called path semantics.

Definition 4 (Formulae of modal μ-calculus)**.** Formulae of the modal μ-calculus *are generated by the following grammar where* $P \in \mathrm{Atom}$, $X \in \mathrm{Var}$, *and the grammar is subject to the side condition that both of* $\mu X.\varphi$ *and* $\nu X.\varphi$ *have no free negative occurrences of* X *in* φ.

$$\begin{array}{rclllllll} \varphi & ::= & P & \mid & \varphi \Rightarrow \varphi & \mid & \Box\varphi & \mid & \Diamond\varphi \\ & \mid & \bot & \mid & \top & \mid & X & & \\ & \mid & \varphi \vee \varphi & \mid & \varphi \wedge \varphi & \mid & \mu X.\varphi & \mid & \nu X.\varphi \end{array}$$

Definition 5 (State semantics)**.** *We define the truth value* $[\![\varphi]\!]_{K,V} \in [S_K, L]$ *of a formula* φ *of modal* μ*-calculus with the signature* Atom *inductively as follows.*

- $[\![P]\!]_{K,V}(s) \overset{\mathrm{def}}{=} \rho_K(s, P)$, *for* $P \in \mathrm{Atom}$
- $[\![\varphi \Rightarrow \psi]\!]_{K,V} \overset{\mathrm{def}}{=} [\![\varphi]\!]_{K,V} \Rightarrow [\![\psi]\!]_{K,V}$
- $[\![\varphi \vee \psi]\!]_{K,V} \overset{\mathrm{def}}{=} [\![\varphi]\!]_{K,V} \vee [\![\psi]\!]_{K,V}$
- $[\![\varphi \wedge \psi]\!]_{K,V} \overset{\mathrm{def}}{=} [\![\varphi]\!]_{K,V} \wedge [\![\psi]\!]_{K,V}$
- $[\![\bot]\!]_{K,V} \overset{\mathrm{def}}{=} \bot$
- $[\![\top]\!]_{K,V} \overset{\mathrm{def}}{=} \top$
- $[\![X]\!]_{K,V} \overset{\mathrm{def}}{=} V(X)$
- $[\![\mu X.\varphi]\!]_{K,V} \overset{\mathrm{def}}{=} \bigwedge\{\, W \mid [\![\varphi]\!]_{K,V[X\mapsto W]} \leq W \,\}$
- $[\![\nu X.\varphi]\!]_{K,V} \overset{\mathrm{def}}{=} \bigvee\{\, W \mid W \leq [\![\varphi]\!]_{K,V[X\mapsto W]} \}$
- $[\![\Box\varphi]\!]_{K,V}(s) \overset{\mathrm{def}}{=} \bigwedge\{\, (s \rightarrow_K t) \Rightarrow [\![\varphi]\!]_{K,V}(t) \mid t \in S_K \,\}$
- $[\![\Diamond\varphi]\!]_{K,V}(s) \overset{\mathrm{def}}{=} \bigvee\{\, (s \rightarrow_K t) \wedge [\![\varphi]\!]_{K,V}(t) \mid t \in S_K \,\}$

As L is a frame, this definition of truth values gives an intuitionistic interpretation. However, when L would moreover be a complete Boolean algebra, i.e., if $\big[(x \Rightarrow \bot) \Rightarrow \bot\big] = x$ for each x, then this automatically would give a classical interpretation, i.e., double negation could be eliminated and the de Morgan duality would hold.

An analogue to the simulation theorem holds, even if Kripke structures are replaced by L-Kripke structures. The usual simulation theorem holds only for a subclass of formulae [6], and it still is true in our case.

Definition 6 ($\Box L\mu$ formulae and $\Diamond L\mu$ formulae)**.** $\Box L\mu$ formulae *are formulae of modal* μ*-calculus with no negative occurrences of subformulae of the form* $\Box\psi$ *and no positive occurrences of subformulae of the form* $\Diamond\psi$. $\Diamond L\mu$ formulae *are defined dually.*

Proposition 7. *Let A and C be L-Kripke structures* Σ *be a simulation from A to C, and* V_A *and* V_C*, respectively, be a path valuation of A and C, respectively. Every* $\Box L\mu$ *formula* ψ *satisfies*

$\Sigma \circ [\![\psi]\!]_{A,V_A} \leq [\![\psi]\!]_{C,V_C}$ *if every variable* X *with some free positive occurrences in* ψ *satisfies* $\Sigma \circ V_A(X) \leq V_C(X)$ *and every variable* X *with some free negative occurrences in* ψ *satisfies* $\Sigma^\circ \circ V_C(X) \leq V_A(X)$*. Every* $\Diamond L\mu$ *formula* ψ *satisfies* $\Sigma^\circ \circ [\![\psi]\!]_{C,V_C} \leq [\![\psi]\!]_{A,V_A}$ *if every variable* X *with some free positive occurrences in* ψ *satisfies* $\Sigma^\circ \circ V_C(X) \leq V_A(X)$ *and every variable* X *with some free negative occurrences in* ψ *satisfies* $\Sigma \circ V_A(X) \leq V_C(X)$*.*

Proof. It is proved by simultaneous induction on ψ. □

Theorem 8 (Simulation theorem in state semantics)**.** *Let* φ *be a closed* $\Box L\mu$ *formula,* A *and* C *be* L*-Kripke structures* Σ *be a simulation from* A *to* C*, and* V_A *and* V_C*, respectively, be a path valuation of* A *and* C*, respectively. Then,* $\Sigma \circ [\![\varphi]\!]_{A,V_A} \leq [\![\varphi]\!]_{C,V_C}$*.*

Proof. It is a corollary of the previous proposition. □

The case $L = 2$, the frame with two values, reduces to the standard simulation theorem.

5 Path Semantics

Linear-time temporal logic (LTL)[5] and computational tree logic (CTL)[5] are temporal logics intensively used for model-checking. While the formulae of CTL and those of modal μ-calculus are *state formulae* which are interpreted as sets of states (state semantics), the formulae of LTL are *path formulae* which are interpreted as sets of paths (path semantics). In this section, we introduce a variant of linear-time modal μ-calculus [7] and define path semantics with respect to an L-Kripke structure. We then prove the simulation theorem for linear-time modal μ-calculus under some condition of L.

A *path* in an L-Kripke structure $(S, \to, \rho)$ is a countable sequence of states. In the context of ordinary Kripke structures, a countable sequence of states is defined to be a path, if each of its states is related to the next state by the transition: $\sigma(n) \to \sigma(n+1) = \top$. In our context, however, the transition may have intermediate degree. It is not natural to regard only the sequences related by $\top$ as real paths. Instead, we define $\mathrm{Weight}(K) \in [[\omega, S], L]$ by $\mathrm{Weight}(K)(\sigma) \stackrel{\mathrm{def}}{=} \bigwedge_{n<\omega}(\sigma(n) \to \sigma(n+1))$.

Given an L-Kripke structure, one can construct another L-Kripke structure by taking the paths of the original structure.

Definition 9 (Path construction)**.** *Let* L *be a frame and* $K = (S, \to, \rho)$ *be an* L*-Kripke structure. We define an* L*-Kripke structure* $\mathrm{Path}(K) = (S', \to', \rho')$ *as follows:*

- $S' \stackrel{\mathrm{def}}{=} [\omega, S]$*, i.e., elements of* S' *are countable sequences of elements of* S*.*
- $\sigma \to' \tau \stackrel{\mathrm{def}}{=} \begin{cases} \top & \text{if } \tau(n) = \sigma(n+1) \\ & \text{for all } n < \omega, \\ \bot & \text{otherwise.} \end{cases}$
- $\rho'(\sigma) \stackrel{\mathrm{def}}{=} \rho(\sigma(0))$.

$[\omega, \Sigma]$: $[\omega, S_A] \nrightarrow [\omega, S_C]$ is defined to be a natural, elementwise extension of Σ: $S_A \nrightarrow S_C$ to countable sequences. Let Head_A: $[\omega, S_A] \nrightarrow S_A$ assign $\top$ to the pairs of the path and the first element and assign $\bot$ to the other pairs.

We define the class of frames that have the sufficient structure to satisfy the simulation theorem in path semantics.

Definition 10 (path-extendable)**.** *A frame* L *is* path-extendable *if* $(\mathrm{Head}_C^\circ \circ I_C) \wedge \mathrm{Weight}(C) \leq [\omega, \Sigma] \circ ((\mathrm{Head}_A^\circ \circ I_A) \wedge \mathrm{Weight}(A))$ *for* L*-relations* Σ: $S_A \nrightarrow S_C$*,* $\to_A$: $S_A \nrightarrow S_A$*,* $\to_C$: $S_C \nrightarrow S_C$*,* $I_C \in [S_C, L]$*, and* $I_A \in [S_A, L]$ *satisfying* $I_C \leq \Sigma \circ I_A$ *and* $\to_C \circ \Sigma \leq \Sigma \circ \to_A$*.*

When $L = 2$, L-relation $(\mathrm{Head}_C^\circ \circ I_C) \wedge \mathrm{Weight}(C)$ corresponds to the set of real paths starting from states of I_C. The frame 2 is path-extendable, since the simulation Σ from A to C which is also a surjective relation to I_C from I_A extends to the pointwise relation $[\omega, \Sigma]$ which is surjective from the set of real paths starting from states of I_A. Similarly, the pointwise frame $L = 2^n$ is also path-extendable for each natural number n.

Not all frames are path-extendable. $L = \omega + 1 = \omega \cup \{\omega\}$ is one of the counterexamples. Let $L = \omega + 1$, $S_C = \{*\}$, $\to_C = \top$, $I_C = \top$, $S_A = \omega + 1$, $I_A = \top$, $\Sigma(x, *) = x$, and $a \to_A b = \top$ if $a > b$ and $a \to_A b =$

$\bot$ otherwise. Then, they satisfy $I_C \leq \Sigma \circ I_A$ and $\rightarrow_C \circ\Sigma \leq \Sigma\circ \rightarrow_A$, but $(\mathrm{Head}_C^\circ \circ I_C) \wedge \mathrm{Weight}(C) = \top$ and $[\omega, \Sigma] \circ ((\mathrm{Head}_A^\circ \circ I_A) \wedge \mathrm{Weight}(A)) = \bot$.

Path semantics can be given only to a subset of modal μ-calculus, called linear-time modal μ-calculus.

Definition 11 (Formulae of linear-time modal μ-calculus)**.** Formulae of linear-time modal μ-calculus *are those of modal μ-calculus without the $\Box$ operator.*

Given a linear-time modal μ-calculus formula, its truth value is defined in terms of state semantics and $\mathrm{Path}(K)$ as follows.

Definition 12 (Path Semantics)**.** *Let φ be a formula in linear-time modal μ-calculus, K be an L-Kripke structure, σ be a path in K, and V be a function from* Var *to $[[\omega, S], L]$. The truth value $\{\!|\varphi|\!\}$ of φ under path semantics in K with V is defined as follows:*

$$\{\!|\varphi|\!\}_{K,V}(\sigma) \stackrel{\mathrm{def}}{=} \mathrm{Weight}(K)(\sigma) \Rightarrow [\![\varphi]\!]_{\mathrm{Path}(K),V}(\sigma)$$

When $L = 2$, the boolean algebra consisting of exactly two values, the truth value under path semantics coincides with the standard path semantics for the standard Kripke structures, provided they are total.

By the definition of $\mathrm{Path}(K)$, every formula of modal μ-calculus satisfies $[\![\Diamond\varphi]\!]_{\mathrm{Path}(K),V} = [\![\Box\varphi]\!]_{\mathrm{Path}(K),V}$. Therefore, the linear-time modal μ-calculus formula can be translated into the $\Box L\mu$ formula that has the same path semantics.

The simulation theorem in path semantics can be formulated as follows.

Theorem 13 (Simulation theorem in path semantics)**.** *Let L be path-extendable, A and C be L-Kripke structures, Σ be a simulation from A to C, and $I_C \in [S_C, L]$ and $I_A \in [S_A, L]$ satisfy $I_C \leq \Sigma\circ I_A$. Let φ be a closed formula in linear-time modal μ-calculus, and V_A and V_C, respectively, be a path valuation of A and C, respectively. Then,* $\mathrm{Head}_C^\circ \circ I_C \leq \{\!|\varphi|\!\}_{C,V_C}$ *provided* $\mathrm{Head}_A^\circ \circ I_A \leq \{\!|\varphi|\!\}_{A,V_A}$.

Proof. The conditions $\rho_C \circ \Sigma \leq \rho_A$ and $\rho_A \circ \Sigma^\circ \leq \rho_C$ imply that $[\omega, \Sigma]$ is a simulation from $\mathrm{Path}(A)$ to $\mathrm{Path}(C)$. By the simulation theorem in state semantics, we have $[\omega, \Sigma] \circ [\![\varphi]\!]_{\mathrm{Path}(A),V_A} \leq [\![\varphi]\!]_{\mathrm{Path}(C),V_C}$. Assume $\mathrm{Head}_A^\circ \circ I_A \leq \{\!|\varphi|\!\}_{A,V_A}$. Then, $[\omega, \Sigma] \circ ((\mathrm{Head}_A^\circ \circ I_A) \wedge \mathrm{Weight}(A)) \leq [\![\varphi]\!]_{\mathrm{Path}(C),V_C}$ holds. Since L is path-extendable, we have $(\mathrm{Head}_C^\circ \circ I_C) \wedge \mathrm{Weight}(C) \leq [\![\varphi]\!]_{\mathrm{Path}(C),V_C}$. That is logically equivalent to $\mathrm{Head}_C^\circ \circ I_C \leq \{\!|\varphi|\!\}_{C,V_C}$. □

6 An Application to Refinement of Models

Model checking of L-Kripke structures for $L = 2^n$ is regarded as multi-valued model checking. This section explains how to apply it to refinement of standard (non-weighted) Kripke structures.

Assume we are to verify some information processing system and construct a standard Kripke structure M which reflects it, set up a formula φ which corresponds to the property to be verified and examine whether $M \models \varphi$.

On the one hand, if $M \models \varphi$ does not hold, then there is (or there often is, at least) a counterexample and we can proceed by analyzing it. On the other hand, $M \models \varphi$ turns out to hold, we cannot conclude the system satisfies φ, because M may not properly reflect the system to be examined. It might be that M does not reflect the original system, and the original system does not satisfy φ, but $M \models \varphi$ holds. In other words, $M \models \varphi$ holds as a result of multiple errors.

So, let D_φ be the set of all (standard) Kripke structures M' which share the state set S and labeling function with M such that $M' \models \varphi$. Consider the partial order on this set defined by

$$N \leq N' \iff \mathrm{id}_S \text{ is a simulation from } N \text{ to } N'.$$

Take all Kripke structures which are larger than M and maximal under this order; that is, take those M' such that $M \leq M'$ and

$$(\forall N) M' \leq N \Rightarrow N = M'.$$

One might hope that by observing those M' it might be judged more easily whether it reflects the original system, because these M' are at "boundary" of Kripke structures which validates φ.

To enumerate these M', one can prepare Kripke structures M_1, M_2, …, M_n by changing only the transition relation of M, and compute

$$\langle [\![M_1 \models \phi]\!], [\![M_2 \models \phi]\!], \ldots, [\![M_n \models \phi]\!] \rangle$$

by multi-valued model checking. The paper [8] is based on this approach.

7 Conclusion

Following the allegory **Mat**(L)[3], we defined weighted relations, weighted Kripke structures, simulations, state semantics of modal μ-calculus, path semantics of linear-time modal μ-calculus. We proved that all frames satisfy the simulation theorem in state semantics, however, not all frames do it in path semantics. Deep analysis of frames that satisfy the simulation theorem in path semantics is future work.

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