# **Bidirectional Interpretation of XQuery**

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# Abstract

XQuery is a powerful functional language to query XML data. This paper gives a bidirectional interpretation of XQuery to address the problem of updating XML data through materialized XQuery views. We first design an expressive bidirectional transformation language, and then translate XQuery expressions into the code of this language. As a result, an XQuery expression can execute in two directions: in the forward direction, it generates a materialized view from the source XML data; while in the backward direction, it updates the source data by putting back the updates on the view. we have implemented our approach and applied it to some XQuery use cases from a W3C draft, which confirms the practicability of this approach.

*Categories and Subject Descriptors* D.3.2 [*Programming Languages*]: Language Classifications—Specialized application languages

General Terms Languages, Transformations

*Keywords* Bidirectional programming, XQuery, XML, view update problem

# 1. Introduction

XQuery [4] is a powerful functional language designed to query XML data. The role of XQuery to XML is just like that of SQL to relational database tables. However, XQuery still lacks an important feature that SQL has. This feature is *view update* [3, 9, 11], that is, updates on a view can be reflected back to the underlying relational database that makes up this view. In other words, XQuery can generate views from the source XML data, but it cannot propagate view updates back into the source data.

This paper presents a translational semantics for XQuery with a bidirectional transformation language. In this bidirectional language, every program can execute in two directions: in the forward direction, it produces a materialized view from the source data; while in the backward direction, it updates the source data by reflecting back the updates on the view. By this way, every XQuery expression can also execute in two directions, and the backward execution will put the updates on views back into the source data.

The underlying language is inspired by the bidirectional language proposed in [10, 5], which includes a collection of combinators, called *lenses*, for tree transformations. The technique used by

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this language is to define both the forward and backward semantics for each combinator, and the backward semantics is responsible for yielding the updated source data. However, as stated in [10], it is not clear what the limits of bidirectional programming with this technique are, or how expressive the combinators defined in this language could be. For our work, the question becomes whether this technique can be used to define an expressive bidirectional language to interpret XQuery. In this paper, we give a positive answer to this question by designing a bidirectional language that is expressive enough to interpret XQuery. The bidirectional language we designed provides a way of treating the variable binding mechanism in a bidirectional context and defines a set of combinators suitable for constructing and destructing XML data. These features are critical to interpret XQuery. For example, the variable binding mechanism provides the basis for interpreting function calls, for and let expressions in XQuery.

We also design a type system for this bidirectional language. Given a program of this language and the type of source data, the type system can check whether this program is type-correct, and if yes, it also generates the corresponding view type. The soundness property of this type system characterizes both the forward and backward behaviors of well-typed programs. For a type-correct program, its forward execution does not get stuck and will generate the view with the correct type. However, its backward execution probably terminates with a special value fail and fail to update the source data even if the updated view conforms to the view type. This happens when updates on the view contain conflicts or improper insertions. The successful backward execution guarantees the updated source data is valid against the given source type.

For XQuery views, we consider three kinds of updates: modifications to text contents, insertions or deletions of elements. The insertions on views are more tricky to deal with than modifications and deletions. This is because inserted values do not have counterparts in the original source data. Hence, it is difficult to determine the structure of the updated source data without the information derived from the original source data for where and how to put back inserted values. This problem is illustrated more by examples in Section 6. We solve this problem by annotating the language constructs with types, which provide guidance information for putting inserted data back in a reasonable way. We, however, do not need users to annotate programs manually. This is done by the type system.

The property of view updating is generally stated by the condition that after the source data is updated according to an updated view, executing the same query on the updated source data should get the same view as the updated view again [3, 9, 5, 10]. However, this condition is not suitable for the view updating problem of XQuery, which is a quite general functional language. For example, if an XQuery expression creates a view containing several copies of one value from the source data (i.e., the dependency in view [13]), then modifying one copy will violate the above condition even if the value in the source data is correctly updated. This

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Figure 1. An XQuery Expression

is because executing this XQuery expression on the updated source data will generate a different view where all copies of the value become the modified one. In this work, the property of view updating is studied by relating the updates on views with those in the source data. The well-behaved bidirectional programs are required to put all view updates back into the related values in the source data if their backward executions are successful.

The main technical contributions in this paper are summarized as follows.

- We design a bidirectional language, which is expressive enough to interpret XQuery. This language also has a sound type system to check the type correctness of bidirectional programs.
- We define the translational semantics of XQuery by giving the translation rules from XQuery Core to the target bidirectional language.
- We illustrate the difficulties of processing insertions on views and propose a type-directed solution.
- The view updating semantics is defined by relating the updates between the source data and view, which is more suitable for the view updating problem of XQuery.
- Our approach has been implemented. The implementation and some examples on XQuery use cases are available at [1].

The remainder of the paper is organized as follows. Section 2 gives an example to illustrate our motivation. Section 3 defines the bidirectional language. Section 4 interprets XQuery. Section 5 presents the type system. Section 6 discusses the insertion problems and proposes our solution. Section 7 introduces our implementation. Section 8 gives the related work and Section 9 concludes the paper.

# 2. An Example

Our motivation can be explained by the XQuery expression in Figure 1, which is an XQuery use case from the W3C draft [8]. Suppose the file "book.xml" contains the following data:

```
<book>
<title>Data on the Web</title>
<author>Serge</author><author>Peter</author>
<author>Dan Suciu</author>
<section id="intro" difficulty="easy">
<title>Introduction</title>Text ... 
<section>
<title>Audience</title>Text ... 
</section>
<section id="syntaxnew" difficulty="medium">
<title>A Syntax For Data</title>Text ... 
</section>
</section>
<title>A Syntax For Data</title>Text ... 
</section>
```

value	::=	$v \mid S$
v	::=	$str^u \mid \langle tag^u \rangle [S]$
S	::=	() $  v_1,, v_n$
u	::=	non   mod   ins   del



```
\begin{array}{rcl} X & ::= & \operatorname{xid} | \operatorname{xconst} S | \operatorname{xvar} Var | \operatorname{xchild} | \operatorname{xsetcnt} X \\ & & | X_1; X_2 | X_1 | | X_2 | \operatorname{xmap} X | \operatorname{xif} P X_1 X_2 \\ & & | \operatorname{xlet} Var X | \operatorname{xfunapp} fname [X_1, ..., X_n] \end{array}
\begin{array}{rcl} P & ::= & \operatorname{xwithtag} str | \operatorname{xistext} | \operatorname{xiselement} | X \\ G & ::= & \epsilon | G, \operatorname{fun} fname(Var_1, ..., Var_n) = X \end{array}
```



Then, in a bidirectional context, the forward execution of the query in Figure 1 will generate the following view, which is the table-of-contents of the book:

```
<toc>
  <section id="intro">
      <title>Introduction</title>
      <section><title>Audience</title></section>
  </section id="syntaxnew">
      <title>A Syntax For Data</title>
  </section>
  </section>
```

On this view, users can modify titles or id attributes, insert or delete sections. These updates will be put back into the source file "book.xml" automatically by executing backward this query. For example, if we change the id attribute on this view from "intro" into "introduction" and insert a new section after the first section, then after the backward execution of this query the value of the corresponding attribute in the source data is also changed into the same value and the new section will appear between the first and second sections. This example can be found at [1].

# 3. The Bidirectional Language

This section introduces the bidirectional language for interpreting XQuery. The backward semantics of the language in this section does not consider insertions on views.

# 3.1 XML Values

The syntax of XML values is given in Figure 2. An XML value is either a single value v or a sequence S of single values. An empty sequence is written as (). To save space, the end tags of XML elements are omitted and their contents are enclosed by brackets.

Strings or elements are annotated with the flag u, which indicates their updating status. The non flag means the strings or the tags of elements are not modified, otherwise the mod flag should be used. The ins flag is for inserted values, and del for deleted values. In addition, if an element has the ins or del flag, then all strings and elements in its contents also have the same flag.

In this work, deleted values in the updated source data are still kept, but flagged with the del flag. They can be removed easily by an independent procedure like the database trigger, which can take into account some application-specific constraints on the source data when removing values. As an example, for the source data in Section 2, if an element title is indicated by del, then the section element containing it can be removed since a section should have a title.

# 3.2 Syntax

The syntax of this language is defined in Figure 3. In this syntax, *Var* and *fname* represent the variable names and function names, respectively. The metavariable X represents bidirectional transformations. The transformations xconst and xvar correspond to the constant or variable expressions in general programming languages. The transformations xchild and xsetcnt are used to get or set the contents of elements. The transformation  $X_1$ ;  $X_2$  is to execute  $X_1$  and  $X_2$  sequentially with the result of  $X_1$  as the input of  $X_2$ , while the transformation  $X_1 || X_2$  executes  $X_1$  and  $X_2$  independently with their results combined as the view. The constructs xmap, xif, xlet and xfunapp corresponds to the expressions of map, if, let and function application in general functional languages, respectively. *G* includes the globally defined functions. Other language constructs, such as those to deal with element attributes or name spaces, are not presented in this paper.

#### 3.3 Semantics

This language supports variable bindings, so evaluation contexts or environments are needed to maintain the values of variables in both forward and backward executions. The context for forward executions is denoted by C, which maps variables to their values; the context for backward executions is denoted by  $\mathcal{E}$ , which maps variables to pairs of values. Suppose for a variable Var,  $\mathcal{E}(Var)$ = (S, S'). Then, S is the original value of Var, and S' is the updated value of Var during backward executions. The notation  $\mathcal{E}(Var)$ .1 is used for the first component of the pair  $\mathcal{E}(Var)$ , and  $\mathcal{E}(Var).2$  for the second component; the notation  $\mathcal{E}.1$  denotes a new evaluation context, say C', defined as  $Dom(\mathcal{E}) = Dom(\mathcal{C}')$ and  $\forall Var \in Dom(\mathcal{E}), C'(Var) = \mathcal{E}(Var).1$ , where  $Dom(\mathcal{E})$  (or  $Dom(\mathcal{C}')$ ) means the domain of  $\mathcal{E}$  (or  $\mathcal{C}'$ ). These contexts can be processed like stacks. The notation  $\mathcal{C} \oplus [Var \mapsto S]$  denotes a new context where a new binding of variable Var to  $\hat{S}$  is pushed onto the top of C, and similarly for pushing new bindings onto  $\mathcal{E}$ . The notation  $\mathcal{E}[Var \mapsto S]$  means the bound value of the least recent variable Var in  $\mathcal{E}$  is changed to S. When we concern the top variable binding in the context  $\mathcal{E}$ , the notation  $\mathcal{E}_1 \bullet [Var \mapsto (S, S')]$ is used to represent  $\mathcal{E}$ , where  $\mathcal{E}_1$  denotes the bottom part of  $\mathcal{E}$ .

Let V be a sequence of XML values. The forward and backward semantics of each language construct is defined in the forms:

- The forward semantics:  $[\![X]\!]_{\mathcal{C}}(S) = V$ , meaning that applying X to the source S generates the view V under the context C.
- The backward semantics: [[X]]<sub>€</sub>(S, V') = (S', E'), meaning that under the environment E, applying X to the updated view V' and the original source S generates the updated source data S'. In addition, a new environment E' is also generated. The backward execution of X probably fails, and the form [[X]]<sub>€</sub>(S, V') = fail is used for such case.

In what follows, we will define the forward and backward semantics for each language construct in Figure 3.

**Identity transformation:** This transformation keeps the (updated) source data and the (updated) view identical in the both directions. It is just the identity lens in [10] except for the evaluation contexts.

$$\begin{split} \llbracket \texttt{xid} \rrbracket_{\mathcal{C}}(S) &= S \\ \llbracket \texttt{xid} \rrbracket_{\mathcal{E}}(S, V) &= (V, \mathcal{E}) \end{split}$$

**Constant transformation:** This transformation returns a constant view V for any source data in the forward direction and returns the original source data in the backward direction without allowing updates on V. When the special value fail is generated, the being executed program terminates immediately.

$$\begin{aligned} \| \texttt{xconst} V \|_{\mathcal{C}}(S) &= V \\ \| \texttt{xconst} V \|_{\mathcal{E}}(S, V') &= \begin{cases} (S, \mathcal{E}), & \text{if } V = V' \\ \texttt{fail}, & \text{otherwise} \end{cases}$$

This transformation provides a template to implement other non-invertible functions in XQuery, such as the sum and comparison operations. Their backward executions do not update the source data and their views cannot be changed.

**Variable reference:** The forward execution hides the source data S and returns the value of the variable *Var* as the view. In its backward execution, the source data is not changed, and instead the value of the variable *Var* in  $\mathcal{E}$  is updated. The mg operation merges the updates in two values and will be defined later.

$$\begin{aligned} \llbracket \operatorname{xvar} \operatorname{Var} \rrbracket_{\mathcal{C}}(S) &= \mathcal{C}(\operatorname{Var}) \\ \llbracket \operatorname{xvar} \operatorname{Var} \rrbracket_{\mathcal{E}}(S, V') &= (S, \mathcal{E}') \\ \text{where} \\ \mathcal{E}' &= \mathcal{E}[\operatorname{Var} \mapsto (\mathcal{E}(\operatorname{Var}).1, \operatorname{mg}(V', \mathcal{E}(\operatorname{Var}).2))] \end{aligned}$$

**Element destructing:** This transformation corresponds to the child axis in XPath, which is used by XQuery to locate elements and attributes in XML data. It returns the contents of the source element in the forward execution, and replaces the contents with the updated view in the backward execution. If the source data is not an element, then the transformation will get stuck, and similarly for the construct xsetcnt below.

$$\begin{split} & [\![\texttt{xchild}]\!]_{\mathcal{C}}(<\!tag^u\!\!>\!\![S]) &= S \\ & [\![\texttt{xchild}]\!]_{\mathcal{E}}(<\!tag^u\!\!>\!\![S],S') &= (<\!tag^u\!\!>\!\![S'],\mathcal{E}) \end{split}$$

**Element constructing:** The source data of this transformation is also required to be an element. In its forward execution, the contents of the source element are replaced by the result of executing the argument transformation X, and in its backward execution the original contents are restored. The updates on V are reflected back to the tag of the source element and the values of variables in  $\mathcal{E}$ .

$$\begin{aligned} & \llbracket \textbf{xsetcnt} \ X \rrbracket_{\mathcal{C}}(\langle tag^{u} \rangle [S]) &= \langle tag^{u} \rangle \llbracket X \rrbracket_{\mathcal{C}}(()) \end{bmatrix} \\ & \llbracket \textbf{xsetcnt} \ X \rrbracket_{\mathcal{E}}(\langle tag^{u} \rangle [S], \langle tag'^{u'} \rangle [V]) &= (\langle tag'^{u'} \rangle [S], \mathcal{E}') \\ & \text{where} \\ & ((), \mathcal{E}') = \llbracket X \rrbracket_{\mathcal{E}}((), V) \end{aligned}$$

The transformation X takes the empty sequence as its source data. This makes the definition of backward semantics more concise since we need not to consider the updated source data generated by X, which is still the empty sequence. This design idea is also applied in the definitions of the transformations of parallel composition, xlet and xfunapp.

**Sequential composition:** This transformation takes two argument transformations  $X_1$  and  $X_2$  and applies them one by one. This definition is the same as that in [10] except that the definition here takes into account the evaluation contexts. Note that the backward execution of  $X_2$  needs to invoke the forward execution of  $X_1$  to generate the intermediate source data.

$$\begin{split} \llbracket X_1; X_2 \rrbracket_{\mathcal{C}}(S) &= \llbracket X_2 \rrbracket_{\mathcal{C}}(\llbracket X_1 \rrbracket_{\mathcal{C}}(S)) \\ \llbracket X_1; X_2 \rrbracket_{\mathcal{E}}(S, V) &= \llbracket X_1 \rrbracket_{\mathcal{E}'}(S, V') \\ \text{where} \\ (V', \mathcal{E}') &= \llbracket X_2 \rrbracket_{\mathcal{E}}(\llbracket X_1 \rrbracket_{\mathcal{E}, 1}(S), V) \end{split}$$

**Parallel composition:** This transformation executes its argument transformations  $X_1$  and  $X_2$  independently, and composes their views in order. The operator len returns the length of a sequence, and the operator  $\texttt{split}(V, [l_1, ..., l_n])$  divides the value V into n subsequences  $V'_i$   $(1 \le i \le n)$ , where  $\texttt{len}(V'_i) = l_i$ . For example,  $\texttt{split}(v_1, v_2, v_3, [2, 1])$  generates two subsequences:  $v_1, v_2$  and

 $v_3$ . For clarity, a sequence value is sometimes underbraced.

$$\begin{split} & \llbracket X_1 \| X_2 \rrbracket_{\mathcal{C}}(S) &= \ \llbracket X_1 \rrbracket_{\mathcal{C}}(()), \llbracket X_2 \rrbracket_{\mathcal{C}}(()) \\ & \llbracket X_1 \| X_2 \rrbracket_{\mathcal{E}}(S, V) &= \ (S, \mathcal{E}'') \\ & \text{where} \\ & V_1', V_2' = \texttt{split}(V, [\texttt{len}(\llbracket X_1 \rrbracket_{\mathcal{E},1}(())), \texttt{len}(\llbracket X_2 \rrbracket_{\mathcal{E},1}(()))]) \\ & ((), \mathcal{E}') = \llbracket X_2 \rrbracket_{\mathcal{E}}((), V_2') \\ & ((), \mathcal{E}'') = \llbracket X_1 \rrbracket_{\mathcal{E}'}((), V_1') \end{split}$$

**Mapping:** Suppose  $S = v_1, ..., v_n$ . This transformation applies its argument transformation X to each single value  $v_i(1 \le i \le n)$  in

the source data S.  

$$\begin{bmatrix} xmap X \end{bmatrix}_{\mathcal{C}}(S) = \begin{bmatrix} X \end{bmatrix}_{\mathcal{C}}(v_1), ..., \begin{bmatrix} X \end{bmatrix}_{\mathcal{C}}(v_n) \\
\begin{bmatrix} xmap X \end{bmatrix}_{\mathcal{E}}(S, V) = \underbrace{(v'_1, ..., v'_n, \mathcal{E}')}_{(v'_1, ..., V'_n = \mathsf{split}(V, [\mathsf{len}(\llbracket X \rrbracket_{\mathcal{E}.1}(v_1)), ..., \mathsf{len}(\llbracket X \rrbracket_{\mathcal{E}.1}(v_n))]) \\
(v'_n, \mathcal{E}_{n-1}) = \begin{bmatrix} X \end{bmatrix}_{\mathcal{E}}(v_n, V'_n) \\
(v'_{n-1}, \mathcal{E}_{n-2}) = \begin{bmatrix} X \rrbracket_{\mathcal{E}_{n-1}}(v_{n-1}, V'_{n-1}) \\
... \\
(v'_1, \mathcal{E}') = \llbracket X \rrbracket_{\mathcal{E}_1}(v_1, V'_1)
\end{bmatrix}$$

**Conditional transformation:** The argument transformation  $X_1$ is chosen if the predicate P holds, otherwise  $X_2$  is chosen. A predicate holds if it does not return the empty sequence.

$$\begin{split} \| \mathtt{xif} \ P \ X_1 \ X_2 \|_{\mathcal{C}}(S) &= \begin{cases} \ \| X_1 \|_{\mathcal{C}}(S), & \text{if} \ \| P \|_{\mathcal{C}}(S) \neq () \\ \| X_2 \|_{\mathcal{C}}(S), & \text{otherwise} \end{cases} \\ \| \mathtt{xif} \ P \ X_1 \ X_2 \|_{\mathcal{E}}(S, V) &= \begin{cases} \ \| X_1 \|_{\mathcal{E}}(S, V), & \text{if} \ \| P \|_{\mathcal{E},1}(S) \neq () \\ \| X_2 \|_{\mathcal{E}}(S, V), & \text{otherwise} \end{cases} \end{cases}$$

**Predicates:** Predicates are only used as the condition of xif, where only their forward semantics are concerned. This means that the predicates are not essential to the expressiveness of our language, and the language can include other needed predicates, such as the existential predicate in XQuery Core, without affecting the definition of xif. The predicates used in this paper will be introduced informally.

The predicate xwithtag str holds if the input data is an element with the tag str; the predicates xiselement and xistext judge whether the input data is an element or a string, respectively. When these three predicates hold, they can return any nonempty value as their results. We let them return the string "true". A transformation X can also be used as a predicate, and its value is determined by its forward semantics.

Variable binding: This construct provides the primitive variable binding mechanism for this bidirectional language. It will be used to define other constructs that need bound variables, such as function calls, and the let and for expressions in XQuery.

$$\begin{aligned} \begin{bmatrix} \text{xlet } Var \ X \end{bmatrix}_{\mathcal{C}}(S) &= \llbracket X \rrbracket_{\mathcal{C} \oplus [Var \mapsto S]}(()) \\ \text{xlet } Var \ X \rrbracket_{\mathcal{E}}(S, V) &= (S', \mathcal{E}') \\ \text{where} \\ ((), \mathcal{E}' \bullet [Var \mapsto (S, S')]) = \llbracket X \rrbracket_{\mathcal{E} \oplus [Var \mapsto (S, S)]}((), V) \end{aligned}$$

The forward semantics of this construct is the same as that of the let in general functional programming languages. Its backward semantics is defined by executing backward the transformation Xunder the context  $\mathcal{E} \oplus [Var \mapsto (S, S)]$ , where the variable Var is bound to a pair of the original source data S. After the backward execution of X, the generated context  $\mathcal{E}' \bullet [Var \mapsto (S, S')]$ contains the updated source data S' in its top binding.

Function call: Suppose the function *fname* is defined as

$$fun fname(Var_1, ..., Var_n) = X$$

Then, the semantics of applying the function *fname* to *n* arguments  $X_1, ..., X_n$  can be defined by using the constructs defined before.

$$\begin{array}{lll} \texttt{xfunapp} \ fname \ [X_1, ..., X_n] &=& \texttt{xconst} \ (); X_1' \\ \texttt{where} \\ X_1' &= X_1; \texttt{xlet} \ Var_1 \ X_2' \\ X_2' &= X_2; \texttt{xlet} \ Var_2 \ X_3' \\ ... \\ X_n' &= X_n; \texttt{xlet} \ Var_n \ X \end{array}$$

In this definition, all argument transformations are first evaluated, and then their results are bound to the corresponding variables. And then, the function body X is executed. Note that in this definition, the source data for the function body is always the empty sequence () due to the definition of xlet. That is, it cannot directly use and update the source data of the transformation

```
fun toc($book-or-section) =
  xvar $book-or-section; xchild;
 xmap (xif (xwithtag ''section'') X0 (xconst ()))
where
  X0 = xlet $section
            (xconst <section>[]; xsetcnt (X1||X2))
  X1 = xvar $section; xchild;
       xmap (xif (xwithtag ''title'') xid (xconst ()))
  X2 = xfunapp toc [xvar $section]
```



xfunapp. Hence, any data to be processed by the function body should be passed as the arguments of the function call.

### 3.4 Merging Updates

For view updating of XQuery, it is common that one source value has several replicas, which may contain different updates. The merging operation mg is to combine all updates in two replicas if there are no conflicts. For example, merging the elements cas in there are no commets. For example, integring the elements
<Title<sup>mod</sup>>[Xquery<sup>mod</sup>] and <title<sup>non</sup>>[XQuery<sup>mod</sup>] will generate
a new element <Title<sup>mod</sup>>[XQuery<sup>mod</sup>], and merging the elements
<price<sup>non</sup>>[30<sup>mod</sup>] and <price<sup>non</sup>>[25<sup>mod</sup>] will cause a conflict.

The mg operation in this section only merges the values without insertions. It will be extended in Section 6 to consider inserted values. This operation is defined as follows.

$$\begin{array}{l} \mathtt{mg}((),())=()\\ \mathtt{mg}(v,S,v',S')=\mathtt{mg}'(v,v'),\mathtt{mg}(S,S') \end{array}$$

The operation mg' merges two strings or elements, defined as follows. 1.7

$$\mathsf{mg}'(str^u, str'^{u'}) = \begin{cases} str^u, \text{if } u \neq \texttt{non and } u' = \texttt{non} \\ str'^{u'}, \text{if } u = \texttt{non and } u' \neq \texttt{non} \\ str^u, \text{if } u = u' \text{ and } str = str' \\ \texttt{fail, otherwise} \\ \mathsf{mg}'(<\!tag^u\!\!>\![S], <\!tag'^{u'}\!\!>\![S']) = <\!tag''^{u''}\!\!>\![S''] \\ \text{where } S'' = \mathsf{mg}(S, S') \\ tag''^{u''} = \mathsf{mg}'(tag^u, tag'^{u'}) \end{cases}$$

The mg' operation fails if the updates of two strings are conflicting or two elements contain conflicting updates on tags or some text contents. In this case, the backward execution terminates immediately with the special value fail.

### 3.5 Programming Examples

To help understand this language, we give two programming examples in this section. The first example uses this language to implement the recursive toc function in Figure 1, which is divided into several pieces for the convenience of reading. The program is given in Figure 4. The function body first gets the contents of the input element. The contents consist of the author, title, section and other elements. Next, only section elements are chosen, and for each section element, the code X0 is used to construct the section element in the view with the help of code X1 and X2, which correspond to the expression \$section/title and the recursive function call in the example query, respectively. The id attribute is omitted in this implementation. It is similar to the code X1 except that the construct xchild should be replaced by xattribute in our implementation.

The child axis of XPath is primitively defined by xchild in the bidirectional language. In the second example, we define another useful axis in XPath, the descendant axis. This axis returns all descendant nodes of the input element. The function xdes below is for this axis in the bidirectional language. It is not difficult to define other XPath axes, such as descendant-or-self, in this language.

```
fun xdes($elm) =
    xvar $elm; xchild;
    xlet $cnt (xvar $cnt || (xvar $cnt; X))
where
    X = xmap (xlet $cnt1 (xfunapp xdes [xvar $cnt1]))
```

#### 3.6 Property of Bidirectional Execution

For the language in this section, a successful backward execution will yield the updated source data which contains all modifications or deletions made on views. To state this property precisely, we assume that all strings and element tags in the source data are annotated with unique identifiers id; all strings and element tags in the arguments of xconst (or the results of non-invertible functions) have the special identifier c. A single value with the identifier Iis written as  $str_I^u$  or  $\langle tag_I^u \rangle [S]$ , where S is also annotated with appropriate identifiers. The identifiers are kept unchanged during transformations and updating views.

Based on these assumptions, the views produced by forward executions also contain strings or elements annotated with identifiers. If a value has the identifier c, then it origins from the arguments of xconst; if it has the identifier *id*, then it comes from the source data.

Suppose S is the original source data or view, S' the updated source data or view. The operation  $S \triangleright S' \Rightarrow U$  returns all updates U in S' with respect to S. The updates in the set U has the form (I, str, str', mod) (or (I, tag, tag', mod)) meaning that the string str (or the tag tag) with identifier I is modified to str (or tag'), or the form (I, del) meaning that the string or element with identifier I is deleted.

$$\begin{split} str_{I}^{\mathrm{non}} \rhd str_{I}^{\mathrm{non}} &\Rightarrow \phi \\ str_{I}^{\mathrm{non}} \rhd str_{I}^{\mathrm{mod}} &\Rightarrow \{(I, str, str', \mathrm{mod})\} \\ str_{I}^{\mathrm{non}} \rhd str_{I}^{\mathrm{del}} &\Rightarrow \{(I, \mathrm{del})\} \\ \frac{S \rhd S' \Rightarrow U}{\langle tag_{I}^{\mathrm{non}} \succ [S] \vDash \langle tag_{I}^{\mathrm{non}} \succ [S'] \Rightarrow U} \\ \hline S \rhd S' \Rightarrow U \\ \hline \langle tag_{I}^{\mathrm{non}} \succ [S] \rhd \langle tag_{I}^{\mathrm{mod}} \succ [S'] \Rightarrow \{(I, tag, tag', \mathrm{mod})\} \cup U \\ \hline \frac{S \rhd S' \Rightarrow U}{\langle tag_{I}^{\mathrm{non}} \succ [S] \rhd \langle tag_{I}^{\mathrm{del}} \succ [S'] \Rightarrow \{(I, \mathrm{del})\} \cup U \\ \hline \frac{v_i \rhd v'_i \Rightarrow U_i \quad (1 \le i \le n)}{v_1, ..., v_n \rhd v'_1, ..., v'_n \Rightarrow U_1 \cup ... \cup U_n} \end{split}$$

Note that the above operation  $\triangleright$  requires users to annotate all view updates correctly: the modified strings or tags must be annotated with the mod flag; the values with del flag cannot be modified at the same time. For view updates with incorrect annotations, this operation will fail since no rule above can be applied. Though this operation is defined here for studying the property of the bidirectional language, it is also used in our implementation to do the sanity check of updated views before backward execution. The property of bidirectional execution is stated below.

THEOREM 1 (Property of Bidirectional Execution). Suppose X is a bidirectional program, S is the source data and  $\phi$  is the empty context C or E. If  $[\![X]\!]_{\phi}(S) = V, V \rhd V' \Rightarrow U_v$  and  $[\![X]\!]_{\phi}(S, V')$  $= (S', \mathcal{E}')$ , then  $\mathcal{E}' = \phi$  and  $U_s = U_v$ , where  $S \rhd S' \Rightarrow U_s$ .  $\Box$ 

This theorem can be proved by induction over each language construct with the help of the following two lemmas: LEMMA 2 is used when proving the construct xvar, which depends on the mg operation in its backward semantics; LEMMA 3 is used when proving those constructs, such as xlet and xsetcnt, which update their contexts after backward executions.

Var	::=	NCName
Expr	::=	$String \mid () \mid Expr, Expr \mid \$Var$
		for \$Var in Expr return Expr
		let $Var := Expr$ return $Expr$
		if (Expr) then Expr else Expr
		Expr op Expr   Axis NodeTest
		element NCName {Expr}
		$ NCName(Expr_1,,Expr_n) $
op	::=	+   <   =   >
Axis	::=	child ::   descendant ::   self ::
NodeTest	::=	$NCName \mid * \mid \texttt{text}() \mid \texttt{node}()$
FunDec	::=	<pre>function NCName(ArgList){Expr}</pre>
ArgList	::=	$Var_1,, Var_n$

Figure 5. Syntax of the XQuery Core

LEMMA 2 Suppose S is the source data, and  $S_1$  and  $S_2$  are two updated replicas of S. If  $S \triangleright S_1 \Rightarrow U_1, S \triangleright S_2 \Rightarrow U_2, S' = mg(S_1, S_2)$  and  $S' \neq \texttt{fail}$ , then  $U_1 \cup U_2 = U_{s'}$ , where  $S \triangleright S' \Rightarrow U_{s'}$ .

LEMMA 3 Suppose X is a bidirectional program, S is the source data and  $\mathcal{E}$  is a evaluation context where each variable is mapped to a pair of same values. If  $[\![X]\!]_{\mathcal{E},1}(S) = V, V \triangleright V' \Rightarrow U_v$ and  $[\![X]\!]_{\mathcal{E}}(S, V') = (S', \mathcal{E}')$ , then  $Dom(\mathcal{E}) = Dom(\mathcal{E}')$  and  $U_{\mathcal{E}} \cup U_s = U_v$ , where  $S \triangleright S' \Rightarrow U_s$ , and  $U_{\mathcal{E}}$  is the union of all U in the set  $\{U|\mathcal{E}'(Var).1 \triangleright \mathcal{E}'(Var).2 \Rightarrow U, Var \in Dom(\mathcal{E}')\}$ .  $\Box$ 

# 4. Interpreting XQuery

The expressions of XQuery can be normalized to the equivalent expressions in XQuery Core, for instance, by the Galax XQuery engine [2]. The syntax of XQuery core is more compact. Hence, like the work [15], we implement bidirectional XQuery based on the XQuery Core syntax.

#### 4.1 Syntax of XQuery Core

The syntax of the XQuery Core presented in this paper is given in Figure 5. In this syntax, the XPath axes, child, descendant and self, implicitly use the reserved variable dot to refer to their context nodes. This syntax does not include the reverse axes of XPath, such as the parent axis. This axis returns the parent of the current context node. Actually, it is difficult to implement reverse axes using the technique in the previous section since from the source element we have no information about its parent node or its ancestor node. But this is not a limitation to our approach. The technique in [16] can be used to rewrite path expressions with reverse axes into equivalent reverse-axis-free ones.

XQuery also includes a lot of predefined functions, such as fn:data and fn:subsequence. In order to process all XQuery expressions, we must define the bidirectional versions of these functions in the underlying bidirectional language. The functions fn:data and fn:subsequence have been supported in our implementation. Although we have not implemented all predefined functions in XQuery, we believe that it is possible to achieve this goal. The basic idea is that if a function is invertible, then the backward semantics is defined to put updates on views back, otherwise its backward semantics does not change the original source data, just like that of the xconst transformation.

#### 4.2 The Translation

Figure 6 gives the rules for translating XQuery Core expressions into the code of the bidirectional language. With such an interpretation, XQuery Core expressions can also execute in two directions: generating the view in the forward direction and putting view updates back in the backward direction. The translation is not diffi-

$[String]_{\mathcal{I}}$	=	xconst String <sup>non</sup>
$[()]_{\mathcal{I}}$	=	xconst()
$\llbracket Expr_1, Expr_2 \rrbracket_{\mathcal{I}}$	=	$\llbracket Expr_1 \rrbracket_{\mathcal{I}}   \llbracket Expr_2 \rrbracket_{\mathcal{I}}$
$\ $ $Var \ _{\mathcal{T}}$	=	xvar \$Var
$\llbracket for \$Var in Expr_1 return Expr_2 \rrbracket_{\mathcal{I}}$	=	$\llbracket Expr_1 \rrbracket_{\mathcal{I}}; xmap (xlet \$Var \llbracket Expr_2 \rrbracket_{\mathcal{I}})$
$\llbracket \text{let } \$Var = Expr_1 \text{ in } Expr_2 \rrbracket_{\mathcal{I}}$	=	$\llbracket Expr_1 \rrbracket_{\mathcal{I}}; \texttt{xlet } \$Var \llbracket Expr_2 \rrbracket_{\mathcal{I}}$
$\llbracket if(Expr) then Expr_1 else Expr_2 \rrbracket_{\mathcal{I}}$	=	$\operatorname{xif} \llbracket Expr \rrbracket_{\mathcal{I}} \llbracket Expr_1 \rrbracket_{\mathcal{I}} \llbracket Expr_2 \rrbracket_{\mathcal{I}}$
$\llbracket Expr_1 \ op \ Expr_2 \rrbracket_{\mathcal{I}}$	=	$xop \[Expr_1]_{\mathcal{T}} \[Expr_2]_{\mathcal{T}}$
Axis NodeTest	=	$[Axis]_{\mathcal{I}}; [NodeTest]_{\mathcal{I}}$
$[child ::]_{\mathcal{I}}$	=	xvar \$dot; xchild
$\llbracket$ descendant :: $\rrbracket_{\mathcal{I}}$	=	xfunapp xdes [xvar $dot$ ]
[self ::]] <sub>⊥</sub>	=	xvar $\$dot$
$[NCName]_{\mathcal{I}}$	=	<pre>xmap (xif (xwithtag NCName) xid (xconst ()))</pre>
[*] <i>I</i>	=	<pre>xmap (xif xiselement xid (xconst ()))</pre>
$\llbracket \texttt{text}() \rrbracket_{\mathcal{I}}$	=	<pre>xmap (xif xistext xid (xconst ()))</pre>
$[node()]_{\mathcal{I}}$	=	xid
[element NCName {Expr}]]	=	xconst <ncname<sup>non&gt;[]; xsetcnt [[<i>Expr</i>]]<sub>I</sub></ncname<sup>
$\llbracket NCName (Expr_1,, Expr_n) \rrbracket_{\mathcal{I}}$	=	xfunapp $NCName [\llbracket Expr_1 \rrbracket_{\mathcal{I}},, \llbracket Expr_n \rrbracket_{\mathcal{I}}]$

Figure 6. Translation of XQuery Core Expression

fun

 $\Gamma;($ 

 $\tau ::= a \mid () \mid str \mid \texttt{string} \mid < tag > [\tau] \mid \tau * \mid \tau, \tau \mid \tau \mid \tau \mid \mu a.\tau$ 

# Figure 7. Syntax of Types

cult due to the expressiveness of the target language. Some of these rules are illustrated below.

In the rule of for expression, the subexpression  $Expr_1$  is first translated, and then composed with an xmap, which takes an xlet with the arguments the variable \$Var and the translation result of the subexpression  $Expr_2$ . That is, the variable Var is bound to each value in the sequence returned by  $[\![Expr_1]\!]_{\mathcal{I}}$ , and then used in  $\llbracket Expr_2 \rrbracket_{\mathcal{I}}.$ 

The operator xop represents the sum (+) and the comparison operators (<, = and >) in the bidirectional language. They are all defined in a style similar to xconst.

In the XQuery Core, the expression Axis NodeTest means the axis Axis first produces a list of nodes from its context node, and then from this list the node test NodeTest selects the nodes satisfying some condition. In the translation of this expression, we need to explicitly get the context node of an axis by referring to the value of the reserved variable \$dot, and then the translation results of Axis and NodeTest are composed.

An XQuery function declaration of the form:

function  $NCName(\$Var_1, ..., \$Var_n) \{Expr\}$ 

is translated into the following function declaration in the bidirectional language:

 $fun NCName(\$Var_1, ..., \$Var_n) = \llbracket Expr \rrbracket_{\mathcal{I}}$ 

The translation defined in Figure 6 satisfies the following property, which says that the translation preserves the semantics of XQuery Core.

THEOREM 4 (Correctness of Translation). Let C be a context that maps variables to XML values. If an XQuery Core expression Expr is evaluated to a value under C, then the expression  $\llbracket Expr \rrbracket_{\mathcal{I}} \rrbracket_{\mathcal{C}}(())$ is also evaluated to the same value.

This theorem can be proved by induction over each translation rule.

#### 5. The Type System

This type system serves two purposes. The first is, as usual, to guarantee the bidirectional programs are type-correct. For example, the xchild cannot be applied to a text node. The second is to annotate

$\Gamma;  au \vdash \texttt{xid}:  au \Rightarrow \texttt{xid}$
$\overline{\Gamma;\tau\vdash\texttt{xconst}\;S:S\Rightarrow\texttt{xconst}\;S}$
$ au' = \Gamma(Var)$
$\overline{\Gamma; \tau \vdash \mathtt{xvar} \ Var: \tau' \Rightarrow \mathtt{xvar}^{\tau'} \ Var}$
$\tau = {{{{\rm{<}}}tag_1}{\rm{>}}[{\tau _1}]  {{\rm{<}}}tag_n}{\rm{>}}[{\tau _n}]$
$\overline{\Gamma;\tau\vdash\texttt{xchild}:\tau_1  \tau_n\Rightarrow\texttt{xchild}^\tau}$
$\tau = {{{{<}}tag_1}{{>}}[\tau _1]}  {{{<}}tag_n}{{>}[\tau _n]}  \Gamma ;() \vdash X:\tau' \Rightarrow X'$
$\overline{\Gamma;\tau\vdash \mathtt{xsetcnt}\;X:{<}tag_1{>}[\tau']  {<}tag_n{>}[\tau']\Rightarrow\mathtt{xsetcnt}\;X'$
$\Gamma; \tau \vdash X_1 : \tau_1 \Rightarrow X_1'  \Gamma; \tau_1 \vdash X_2 : \tau_2 \Rightarrow X_2'$
$\Gamma; \tau \vdash X_1; X_2 : \tau_2 \Rightarrow X_1'; X_2'$
$\Gamma; () \vdash X_1 : \tau_1 \Rightarrow X'_1  \Gamma; () \vdash X_2 : \tau_2 \Rightarrow X'_2$
$\Gamma; \tau \vdash X_1    X_2 : \tau_1, \tau_2 \Rightarrow X_1'   _{\tau_1}^{\tau_2} X_2'$
$\Gamma;\tau\vdash_{\mathtt{m}}\mathtt{xmap}\;X:\tau'\Rightarrow\tau'' \Gamma;\tau''\vdash X:\tau'''\Rightarrow X'$
$\Gamma;\tau\vdash\mathtt{xmap}\;X:\mathtt{rmbar}(\tau')\Rightarrow\mathtt{xmap}^{\tau'}\;X'$
$\begin{array}{l} \Gamma; \tau \vdash P : \tau_P \Rightarrow P'  \Gamma; T(\tau, P) \vdash X_1 : \tau_1 \Rightarrow X_1' \\ \Gamma; F(\tau, P) \vdash X_2 : \tau_2 \Rightarrow X_2' \end{array}$
$\Gamma; \tau \vdash \texttt{xif} \ P \ X_1 \ X_2 : \tau_1   \tau_2 \Rightarrow \texttt{xif}_{\tau_1}^{\tau_2} \ P \ X_1' \ X_2'$
$P \in \{\texttt{xiselement}, \texttt{xistext}, \texttt{xwithtag} \ str\}$
$\Gamma; \tau \vdash P: \texttt{string} () \Rightarrow P$
$\Gamma[\operatorname{Var} \mapsto \tau]; () \vdash X : \tau' \Rightarrow X'$
$\overline{\Gamma; \tau \vdash \texttt{xlet } Var \; X: \tau' \Rightarrow \texttt{xlet } Var \; X'}$
$ \begin{aligned} & fun\; fname(Var_1,,Var_n) = X \in G \\ & \Gamma; () \vdash X_i : \tau_i \Rightarrow X'_i \; 1 \leq i \leq n  fname(\tau_1,,\tau_n) \not\in Dom(\Gamma) \\ & [Var_1 \mapsto \tau_1,,Var_n \mapsto \tau_n, \\ & fname(\tau_1,,\tau_n) \mapsto a]; () \vdash X : \tau' \Rightarrow X'  a \text{ is fresh} \end{aligned} $
$\Gamma; \tau \vdash \mathtt{xfunapp} \ fname \ [X_1,, X_n]; \mu \ a.\tau'$
$\Rightarrow \texttt{xfunapp}^{[\tau_1,,\tau_n]} \text{ fname } [X'_1,,X'_n]$
$\begin{array}{l} \texttt{fun } \textit{fname}(Var_1,,Var_n) = X \in G \\ \Gamma; () \vdash X_i : \tau_i \Rightarrow X'_i \ 1 \leq i \leq n  \Gamma(\textit{fname}(\tau_1,,\tau_n)) = a \end{array}$
$\Gamma: \tau \vdash xfunann fname [X_1, X_n]: a$



 $\Rightarrow$  xfunapp<sup>[ $\tau_1,...,\tau_n$ ]</sup> fname  $[X'_1,...,X'_n]$ 

Figure 9. Typing Rules for xmap

some program constructs with types, and the type information will be used to process insertions.

The syntax of types is given in Figure 7, which is just the regular expression types in [12]. As the definition there, the notation  $S \in \tau$  means the value S has the type  $\tau$ . The recursive type  $\mu a.\tau$  is regarded as equivalent to its unfolded form  $\tau[\mu a.\tau/a]$ , where all occurrences of the free type variable a in  $\tau$  are replaced with  $\mu a.\tau$ . For brevity, recursive types and type variables will not be considered in this paper. Note that a string *str* is also a type, containing only itself.

The typing rules for the bidirectional transformation language are defined in Figure 8. The judgment has the form  $\Gamma; \tau \vdash X :$  $\tau' \Rightarrow X'$ , meaning that under the typing context  $\Gamma$ , if the source data has the type  $\tau$ , the transformation X will generate a view with the type  $\tau'$ . The transformation X' is the result of annotating X with types. The typing context  $\Gamma$  maps variables to their types or function names together with the types of their arguments to their view types.

In Figure 8, only those constructs that need type information to process insertions are annotated with types. How to use them will be discussed in the next section. The transformation xvar is annotated with the type of the variable it concerns, xchild is annotated with the type of its source data, and the parallel composition is annotated with the view types of its two argument transformations.

The transformation xmap applies its argument transformation X to each single value in the source data. Therefore, in its typing rule, we need to identify each string and element type in the sourcedata type of xmap, and then use it as the source-data type to check X. The view type of xmap is just its source-data type with each string or element type replaced by the view type of applying Xto it. This typing procedure is defined by rules in Figure 9. These rules also collect all top-level string and element types in the source-data type of xmap and represent them by a choice type. This choice type will then be used to check X again, so that X will be annotated with all possible string or element types. The view type of xmap generated by the rules in Figure 9 contains a special star  $\bar{*}$ , which tells that the type component modified by it corresponds to a string type or element type, which is also modified by \* in the source-data type. The view type with  $\overline{*}$  is only used to annotate xmap, and otherwise  $\bar{*}$  is changed into \* by the operator rmbar. The example below illustrates that the refined \* can help update the source data in a more reasonable way for the inserted values. The use of refined \* is given in Figure 13.

For example, suppose we have the code xmap xchild. If the source-data type is <box>[<apple>[string]\*], then the view type annotated on xmap is <apple>[string]\*; if the source-data type is <box>[<apple>[string]]\*, then the view type annotated on xmap is <apple>[string]]\*. This refined \* can tell us whether

let $P = xwith$	tag $s$	tr
$\mathtt{T}(\tau, P)$	=	(), where $\tau \in \{(), \mathtt{string}, \tau *\}$
$\mathtt{T}({\boldsymbol{<}}tag{\boldsymbol{>}}[\tau],P)$	=	$\begin{cases} \langle tag \rangle [\tau], & \text{if } str = tag \\ (), & \text{otherwise} \end{cases}$
$\mathtt{T}(\underbrace{\tau_1,\tau_2},P)$	=	$\begin{cases} T(\tau_1, P), & \text{if } \tau_2 = () \\ T(\tau_2, P), & \text{if } \tau_1 = () \\ (), & \text{otherwise} \end{cases}$
$\mathtt{T}( au_1  au_2,P)$	=	$\widetilde{T}(\tau_1, P)   \mathtt{T}(\tau_2, P)$

Figure 10. The operator T for xwithtag

the apple elements in a view come from the same box element or from a sequence of different box elements. If we insert a new apple element on the view, which already contains a list of apple elements from the source data, then in the first case, the new apple element should be used together with other existing apple elements by xchild to update the source data, resulting in the new box element containing both the existing apple elements and the new inserted one; while in the second case, the inserted apple element should be processed independently by xchild as other existing elements, and the result is the updated source contains a new box element for this new apple element. Anyway, in both cases, the updated source data still has the valid type due to using the information provided by the refined \*. If both the box and apple type in the source-data type are modified by \*, then xmap will be annotated by  $\langle apple \rangle [string] * \overline{*}$ . For this case, both updating ways, putting the new element back into an existing or a new box element, generate the valid source data. In this work, we choose the first updating way since it leads to less changes on the source data.

The typing rule of xif checks its two branches under the sourcedata type computed by  $T(\tau, P)$  and  $F(\tau, P)$ , respectively. This is to generate more accurate view types for each branch. The following example shows that such accuracy is useful. In this example, suppose we have the code xif (xwithtag "book") xchild (xconst ()) and the source-data type <book>[string]|string. If the source-data type of xif is directly used to check its branches, the true branch will cause a type error since xchild can only be applied to elements. Actually, if the true branch is chosen at runtime, we know the xwithtag predicate must hold, so the source data of this branch must be an element. The operator  $T(\tau, xwithtag str)$ selects in  $\tau$  the element types with the tag *str*, which is defined in Figure 10, and the operator  $F(\tau, xwithtag str)$  does the reverse selection. These two operators on xiselement and xistext are defined similarly. For any transformation X, both  $T(\tau, X)$  and  $F(\tau, X)$  returns  $\tau$ . After type checking, xif is annotated with the view types of its two branches.

There are two typing rules for function calls. If a function together with the types of its arguments is not in the domain of  $\Gamma$ , then the first rule is used, otherwise the second is taken. In the first rule, the function body X is checked under the typing context, where the variable  $Var_i$  is mapped to the type  $\tau_i$ , and the function name funname together with these argument types is mapped to a fresh type variable a. The view type  $\tau'$  of the function body X probably contains the free type variable a because of recursive function calls. Therefore the view type of xfunapp in the first typing rule is a recursive type  $\mu a. \tau'$ . In the second rule, the function body will not be checked since its resulting type is already available. Note that the type-annotated function body in the first rule is not used in the typing result. This does not mean that we do not need type annotations in the function body. Rather, this is because we want to avoid the trouble of managing different versions of the same function with different type annotations. Our approach is to annotate function calls with the types of their arguments, and then use these types to type check and annotate function bodies when meeting with function calls at runtime.

The soundness property of this type system is stated as follows. This property concerns both the forward and backward behaviors of well-typed programs. For the backward behavior, the type system cannot guarantee a well-typed program will not fail since it cannot check conflicting and improperly updates statically.

THEOREM 5 (Soundness). Given a transformation X and a source value S, if  $\phi; \tau \vdash X : \tau' \Rightarrow X'$  and  $S \in \tau$ , then  $[\![X']\!]_{\phi}(S) = V$  and  $V \in \tau'$ ; and moreover, if  $V' \in \tau'$  and  $[\![X']\!]_{\phi}(S,V') = (S', \mathcal{E}')$ , then  $\mathcal{E}' = \phi$  and  $S' \in \tau$ .

Note that from this theorem modifications to element tags are not allowed if they produce views violating the expected types, although they are allowed as updating operations.

# 6. Insertions

This section discusses several view updating problems caused by insertions, and shows our type-based solution. In the solution, values are always needed to be related to the types against which they have been validated, and the related type is consulted when merging or splitting values. For this purpose, we annotate each string and element type with the unique index I, written as string<sub>I</sub> and  $\langle tag_I \rangle [\tau]$ . After an XML value is successfully validated against a type, all strings and elements within this value are annotated with the corresponding indexes in this type. For a validated value S or a type  $\tau$ , the operator Id(S) or Id( $\tau$ ) returns a set of indexes at the top-level of S or  $\tau$ . Hence if Id(S)  $\subseteq$  Id( $\tau$ ), we know the value Shas the value  $\tau$ .

The source data used in this section is shown below. It contains a list of books, each of which may contain a title and any number of authors and the non flags on values are omitted.

<book>[<title>[a], <author>[b]], <book>[<title>[c], <author>[d], <author>[e]]

#### 6.1 Merging Inserted Values

We use the following code to illustrate the problem when merging two inserted values. The variable **\$books** in this expression is supposed to be bound to the above source data.

The view of the above code consists of two items. Consider the following updated view.

Then, during the backward execution of the above code, the expression X1 and X2 will deal with the title and author elements in the view, respectively. For the third item, since it is not generated from a book in the original source, the variable \$b is not bound to any value at the beginning of the backward execution of xlet above. The backward execution for processing the third item is sketched as follows. First, X1 is used to deal with the inserted title element. After its backward execution the value of \$b can be simply set to a new book element containing only the inserted title since \$b has no valid value yet. Next, X2 is used to deal with the inserted author element. When executing the backward execution of xvar in X2, its view is a new book element containing

$$\begin{split} & \operatorname{mg}_{\operatorname{ins}}(S_1,(),\tau) = S_1 \\ & \operatorname{mg}_{\operatorname{ins}}(),S_2,\tau) = S_2 \\ & \operatorname{mg}_{\operatorname{ins}}(S_1,S_2,\tau*) = \underbrace{S_1,S_2} \\ & \operatorname{mg}_{\operatorname{ins}}(str_I^{\operatorname{ins}},str_I'^{\operatorname{ins}},\operatorname{string}_I) = \begin{cases} str_I^{\operatorname{ins}}, & \text{if } str = str' \\ \operatorname{fail}, & \text{otherwise} \end{cases} \\ & \operatorname{mg}_{\operatorname{ins}}(stag_I^{\operatorname{ins}} > [S_1], < tag_I^{\operatorname{ins}} > [S_2], < tag_I > [\tau]) = < tag_I^{\operatorname{ins}} > [S'] \\ & \text{where } S' = \operatorname{mg}_{\operatorname{ins}}(S_1, S_2, \tau) \\ & \operatorname{mg}_{\operatorname{ins}}(\underbrace{S_1,S_1',S_2,S_2',\tau_1,\tau_2}) = \operatorname{mg}_{\operatorname{ins}}(S_1,S_2,\tau_1), \operatorname{mg}_{\operatorname{ins}}(S_1',S_2',\tau_2) \\ & \text{where } \operatorname{Id}(S_1) \subseteq \operatorname{Id}(\tau_1), \operatorname{Id}(S_2) \subseteq \operatorname{Id}(\tau_1), \\ & \operatorname{Id}(S_1') \subseteq \operatorname{Id}(\tau_2) \text{ and } \operatorname{Id}(S_2') \subseteq \operatorname{Id}(\tau_2). \\ & \operatorname{mg}_{\operatorname{ins}}(S_1,S_2,\tau_1|\tau_2) = \\ & \begin{cases} \operatorname{mg}_{\operatorname{ins}}(S_1,S_2,\tau_1), & \text{if } \operatorname{Id}(S_1) \subseteq \tau_1 \text{ and } \operatorname{Id}(S_2) \subseteq \tau_1 \\ & \operatorname{mg}_{\operatorname{ins}}(S_1,S_2,\tau_2), & \text{if } \operatorname{Id}(S_1) \subseteq \tau_2 \text{ and } \operatorname{Id}(S_2) \subseteq \tau_2 \\ & \operatorname{fail}, & \text{otherwise} \end{cases} \end{cases} \end{split}$$

Figure 11. The Operator mg<sub>ins</sub>

only the inserted author element. Obviously, the merging operation used by xvar should merge this view with the existing value of \$b, that is, to build a new book element containing both the inserted title and author elements. In this example, it seems that we can merge these two book elements just by putting their contents together in the order they appear on the view. However, this is probably not true for other cases. For example, the order of contents on a view perhaps is different from that in the source, and contents are perhaps needed to be merged further into a new content.

In our work, the merging operation for inserted values is guided by types, which characterize the structures of the expected merging results. These types can be obtained from the annotations of xvar. This merging operation is defined in Figure 11, which has a type argument except the two values to be merged. Before using the operator  $mg_{ins}$ , the values to be merged should be validated against the type argument, such that all strings and elements in each value can be related to the corresponding type components.

LEMMA 6 Suppose  $S_1 \in \tau$  and  $S_2 \in \tau$ . If  $S_3 = \text{mg}_{\text{ins}}(S_1, S_2, \tau)$ and  $S_3 \neq \text{fail}$ , then  $S_3 \in \tau$ .

As an example, if the expected result of merging  $S_1$  and  $S_2$  is described by  $\tau$ \*, then the merging result will be  $S_1, S_2$ , which has the expected type  $\tau$ \* since  $S_1$  and  $S_2$  have been validated against  $\tau$ \*.

### 6.2 Child Axis and Conditional Transformation

The transformations xchild and xif also have problems when dealing with inserted values. Recall the definitions of their backward semantics, xchild needs the tag of the original element to determine the tag of the updated element, and xif needs the source data to determine which branch transformation should be chosen.

These problems are also solved by using the annotated types. The type on xchild can provide information about what tag the new source element should have, and the types on xif can help choose the branch transformation according to which view type the inserted value has. Note that the element returned by xchild in this way is also annotated with the ins flag, and if the view types annotated on xif are overlapped, the first view type has priority in our implementation. The accuracy policy taken by the typing rule of xif can greatly reduce the possibility of overlap in practice.

#### 6.3 Splitting Inserted Values

The operator split is used by both xmap and the parallel composition to divide their views into subsequences, and then each subsequence is used as a view to perform the backward transformation. When a view does not include inserted values, it can be divided

$$\begin{aligned} & \text{split}((), [], \tau) &= () \\ & \text{split}(str_{I}^{u}, [1], \text{string}_{I}) &= str_{I}^{u}, \text{ if } u \neq \text{ ins}} \\ & \text{split}(str_{I}^{u}, [1], \text{string}_{I}) &= str_{I}^{u} \\ & \text{split}(str_{I}^{u}, [1], str_{I}^{u}, [1], str_{I}^{u}, [1], str_{I}^{u}, [1], str_{I}^{u}, [1$$

Figure 13. The Operator split for xmap

 $split(V, [], [\tau_1, \tau_2]) = V_1, V_2$ where  $isIns(V), Id(V_1) \subseteq Id(\tau_1), Id(V_2) \subseteq Id(\tau_2)$ and  $V_1, V_2 = V$ .  $split(V, [l_1, l_2], [\tau_1, \tau_2]) = V_1, V_2$ where isNotIns(V),  $Id(V_1) \subseteq Id(\tau_1)$ ,  $Id(V_2) \subseteq Id(\tau_2)$ ,  $V_1, V_2 = V$  and the following P holds for  $1 \le i \le 2$ .  $P = \begin{cases} \texttt{lenNoIns}(V_i) = l_i, & \text{if } l_i > 0\\ V_i = (), & \text{if } l_i = 0 \end{cases}$  $\operatorname{split}(V, ls, [\tau_1, \tau_2]) = \operatorname{fail}$ , if no above case can be applied.

s



precisely according to the expected length for each subsequence computed from the original source data. When the view includes inserted values, the length information tells nothing about how to divide them. The following example illustrates that an elegant splitting mechanism is needed for views with inserted values.

For the above source data, the code xmap xchild produces a view consisting of a sequence of titles and authors of each book. Consider the following updated view.

```
<title>[a], <author>[b], <author<sup>ins</sup>>[], <title<sup>ins</sup>>[],
<author<sup>ins</sup>>[], <title>[c], <author>[d], <author>[e]
```

In the backward execution of code xmap xchild, this view is first divided into subsequences and then each of them is used as the updated view of xchild. For this example, it is reasonable to split the updated view into three subsequences: the first three elements, the next two, and the last three. Thus, in the updated source data, the first book is inserted with a new author, the second book is new and contains the inserted title and author, and the third book is unchanged. This example can also be found at [1].

The revised split for the parallel transformation is given in Figure 12. It takes three arguments: the first is the updated view to be split; the second is a list of integers, each of which indicates the number of values expected by a subsequence; the third one  $[\tau_1, \tau_2]$  contains the view types for the two composed transformations. The first case is applied when V contains only inserted

values, guarded by the predicate isIns(V). In this case, the second argument is an empty list since there is no original source data available to compute this integer list. The second case, guarded by the predicate isNotIns(V), is applied when V contains both the values computed from the original source data and inserted values. The operator  $lenNoIns(V_i)$  returns the length of  $V_i$  without counting inserted values. Before being split, the updated view should be validated against the sequence type  $\tau_1, \tau_2$ . Note that in the second case, if a subsequence is expected to have the zero length, then it must be the empty sequence () and cannot include any inserted values. This is because the zero length means the original source data for this subsequence is hidden by using the code xconst (), which does not accept a changed view. This split operator only produces two subsequences, corresponding to the two composed transformations.

The revised split for xmap is given in Figure 13. Its first argument is the updated view to be split; the second also contains integers for the expected length of each subsequence; the third is the view type of xmap. The splitting procedure is directed by the view type. To illustrate this operator, we take the cases for the type  $\tau_1, \tau_2$  as examples: the first case means V belongs to either  $\tau_1$  or  $\tau_2$ ; the second case is applied when V contains only inserted values; the third case is used to separate the inserted subsequence at the head or the tail of V; otherwise, the fourth case is applied, which either produces an empty subsequence () for each zero integer in the second argument ls, or divides both V and ls into two parts such that the first (or second) part of V belongs to  $\tau_1$  (or  $\tau_2$ ) and its length without counting inserted values should be the same as the sum of integers in the first (or second) part of ls. A type-correct view probably cannot be split sensibly. For example, if a new title is inserted between an original title and its following original authors in the view of code xmap xchild, then splitting this view will fail. This is an example of improper insertions. A view should be validated against with the view type before splitting. Note that when applying the rule for the type  $\tau \bar{*}$ , the indexes on the type  $\tau, \tau\bar{*}$  should be re-assigned such that they are unique among  $\tau$  and

 $\tau \bar{*}$ , and then V needs to be validated against this type to get new

indexes. The number of subsequences produced by this split can be greater than the length of the original source data, but never less than. This means that xmap can insert new values in its source data after backward executions.

# 7. Implementation

The approach proposed in this work has been implemented in Java with JDOM. Our system is available at [1], where several XQuery Core examples are also provided. Most of these examples are obtained by normalizing XQuery use cases from the W3C draft [8] with the Galax XQuery engine [2].

Our implementation supports more XQuery Core syntax than we present in this paper. For example, the order expression in XQuery, the existential predicate, the attribute axis, XML name spaces and the constructors for constructing and destructing sequences (or lists) are supported in our implementation. More interestingly, our implementation can simulate higher-order functions in functional languages by changing the argument *fname* in xfunapp from a string to a transformation, and therefore a function argument can also be used as a function name. This feature is useful when we use this bidirectional language to interpret HaXML [17], which contains some higher-order XML transformation combinators.

In this implementation, only the top node of an inserted or deleted element needs to be marked with ins or del, and other flags are derived by the system automatically. This prototype implementation is not used to benchmark the performance of our approach since the implementation itself can be improved and the code generated from XQuery Core has much space to optimize. In our approach, the values generated by xconst and aggregate functions, such as sum and count, cannot be modified. We review the first forty-one XQuery use cases in [8]. Only six of them generate views completely consisting of values from xconst or aggregate functions, and do not allow any update. For other use cases, our approach can be found useful to enable view update of XQuery.

# 8. Related Work

The main related work can be described from two aspects. The first is related to the bidirectional language design, and the second is about XML view update.

The related work in the first aspect includes [10, 5, 13]. They cannot be used directly to interpret XQuery for the following reasons. First, these languages do not have variable binding mechanisms, and consequently the output of a transformation can only be used by its successive transformations or the transformation combinators containing it. However, in XQuery, an output from an expression may be bound to a variable, and then used many times by different subexpressions. Second, these languages do not provide a general setting to interpret functions in XQuery. A function in XQuery can have any number of arguments, each of which may be used as the updatable source data. However, the current languages only allow functions with one argument as the updatable source data. Third, the constructs in these languages are designed for their particular purposes and are not suitable for interpreting XQuery. For example, XPath axes are difficult to interpret in these languages.

The work [6, 7] studies how to update the relational database through XML views, rather than update XML data like our work. They use query trees to capture common operations in most XML query languages. However, query trees cannot support recursive functions in XQuery, as shown by our motivating example. The work in [14] also uses programming language technique to solve the view updating problem. But their view definition language is not bidirecitonal, so when defining a view, users have to write the code for putting back possible updates into the source XML data.

# 9. Conclusion

In this paper, we design an expressive bidirectional XML transformation language, and then use it to interpret XQuery. We use this approach to address the view updating problem of XQuery. Although we are motivated by interpreting XQuery, we believe that our work provides a potential technique to define general bidirectional functional languages since the bidirectional semantics of functions and some constructors for algebraic data types can be defined in this technique.

This work provides a basis for several future work. The first is to analyze bidirectional programs and tell what are valid updates on views, such that valid updates do not lead to failure during backward executions. The second is to optimize the target language code, for instance, by generating the efficient specialized backward code for a particular source data from the forward execution.

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