

# Program for Class Discussion

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## Problem 2-1:

Show that

$$\{P_0\}S\{Q_0\} \text{ and } \{P_1\}S\{Q_1\}$$

implies

$$\{P_0 \wedge P_1\}S\{Q_0 \wedge Q_1\} \text{ and } \{P_0 \vee P_1\}S\{Q_0 \vee Q_1\}.$$

**Speaker:** 48076134

## Problem 2-2

Prove

```
[[
var  $x, y : int;$ 
 $\{x = A \wedge y = B\}$ 
 $x := x - y; y := x + y; x := y - x$ 
 $\{x = B \wedge y = A\}$ 
]].
```

**Speaker:** 37086868

## Problem 2-3

Determine the weakest  $P$  such that

```
[[
var  $x : int;$ 
 $\{P\}$ 
 $x := x + 1;$ 
if  $x > 0 \rightarrow x := x - 1$ 
   $\square x < 0 \rightarrow x := x + 2$ 
   $\square x = 1 \rightarrow skip$ 
fi
 $\{x \geq 1\}$ 
]].
```

**Speaker:** 48086234, 48086226

**Problem 2-4**

Prove the correctness of the following program.

```

[[
  var x, y, z : int;
  {true}
  do x < y → x := x + 1
    [] y < z → y := y + 1
    [] z < x → z := z + 1
  od
  {x = y = z}
]]

```

**Speaker:** 48086219

**Problem 2-5**

The following problem may be used to compute (non-deterministically) natural numbers  $x$  and  $y$  such that  $x * y = N$ . Prove:

```

[[
  var p, x, y, N : int;
  {N ≥ 1}
  p, x, y := N - 1, 1, 1;
  {N = x * y + p}
  do p ≠ 0
    → if p mod x = 0 → p, y := p - x, y + 1
      [] p mod y = 0 → x, p := x + 1, p - y
    fi
  od
  {x * y = N}
]].

```

**Speaker:** 48086229

**Problem 2-6**

Prove

```

[[
  con N : int {N ≥ 0};
  f : array [0..N) of int;
  var b : bool;
  [[
    var n : int;
    b, n := false, 0;
    do n ≠ N → b := b ∨ f.n = 0; n := n + 1 od
  ]]
  {b ≡ (∃i : 0 ≤ i < N : f.i = 0)}
]].

```

**Speaker:** 37086386

**Problem 3**

Let  $X[0..N)$  be an integer array. Express the following expressions in a natural language.

1.  $b \equiv (\forall i : 0 \leq i < N : X.i \geq 0)$
2.  $r = (\max p, q : 0 \leq p \leq q \leq N \wedge (\forall i : p \leq i < q : X.i \geq 0) : q - p)$
3.  $r = (\#k : 0 \leq k < N : (\forall i : 0 \leq i < k : X.i < X.k))$
4.  $b \equiv (\exists i : 0 < i < N : X.(i - 1) < X.i)$
5.  $r = (\#p, q : 0 \leq p < q < N : X.p = 0 \wedge X.q = 0)$
6.  $s = (\max p, q : 0 \leq p < q < N : X.p + X.q)$
7.  $b \equiv (\forall p, q : 0 \leq p \wedge 0 \leq q \wedge p + q = N - 1 : X.p = X.q)$
8.  $b = (\exists i : 0 \leq i < N.X.i = 0)$

**Speaker:** 48086213

**Problem 4-1**

Solve the following problem.

```

[[
con N, X : int {N ≥ 0}; f : array [0..N) of int;
var r : int
S
{r = (Σ i : 0 ≤ i < N : f.i * Xi)}
]].

```

**Speaker:** 48086216, 48086230

**Problem 4-2**

Solve the following problem.

```

[[
con N : int {N ≥ 1}; A : array [0..N) of int;
var r : int
S
{r = (max p q : 0 ≤ p < q < N : A.p - A.q)}
]].

```

**Speaker:** 48087201

**Problem 5-1**

Solve

```

[[
con N, X : int {N ≥ 0}; f : array [0..N) of int;
var r : bool
S
{r ≡ (∃ i : 0 ≤ i < N : f.i = 0)}
]],

```

by defining for  $0 \leq n \leq N$

$$G.n \equiv (\exists i : n \leq i < N : f.i = 0)$$

and deriving a suitable recurrence relation for  $G$ .

**Speaker:** 48086406

**Problem 5-2**

An  $h$ -sequence is either a sequence consisting of the single element 0 or it is a 1 followed by two  $h$ -sequences. Syntactically,  $h$ -sequence may be defined by

$$h = 0 \mid 1 \ h \ h$$

Solve

```
[[
con  $N : int \{N \geq 0\}$ ;  $A : \mathbf{array} [0..2 * N + 1]$  of  $[0..1]$ ;
var  $r : bool$ ;
 $S$ 
 $\{r \equiv A \text{ is an } h\text{-sequence}\}$ ]].
```

**Speaker:** 48086225

**Problem 5-3**

Derive a program to solve the following problem.

```
[[
con  $N : int \{N \geq 0\}$ ;
 $X, Y, Z, W : \mathbf{array} [0..N]$  of  $int$ ;
var  $r : int$ ;
 $S$ 
 $\{r = \#\{i, j, k, l : 0 \leq i, j, k, l < N : X.i + Y.j + Z.k + W.l = 0\}\}$ 
]].
```

**Speaker:** 48086227

**Problem 6-1**

Derive a program that has time complexity  $\mathcal{O}(\log N)$  for

```
[[
con  $N : int \{N \geq 1\}$ ;  $f : \mathbf{array} [0..N]$  of  $int \{f.0 < f.N\}$ ;
var  $x : int$ ;
 $S$ 
 $\{0 \leq x < N \wedge f.x < f.(x + 1)\}$ 
]].
```

by introducing variable  $y$  and invariants

$$P_0 : f.x < f.y$$

$$P_1 : 0 \leq x < y \leq N$$

**Speaker:** 48086215

**Problem 6-2**

Derive an  $\mathcal{O}(\log N)$  algorithm for *square root*:

```

[[
  con N : int {N ≥ 0};
  var x : int;
  square root
  {x2 ≤ N ∧ (x + 1)2 > N}
]]

```

by introducing variables  $y$  and  $k$  and invariants:

$$P_0 : x^2 \leq N \wedge (x + y)^2 > N$$

$$P_1 : y = 2^k \wedge 0 \leq k$$

**Speaker:** 48086217

**Problem 6-3**

Solve

```

[[
  con A, B, N : int {N ≥ 0};
  var x : int;
  S
  {x = (∑i : 0 ≤ i ≤ N : AN-i * Bi)}
]]

```

**Speaker:** 48067210

**Problem 6-4**

Solve

```

[[
  con N : int {N ≥ 0};
  var x : int;
  Fibolucci
  {x = (∑i : 0 ≤ i ≤ N : fib.i * fib.(N - i))}
]]

```

where *fib* is defined by

$$fib.0 = 0$$

$$fib.1 = 1$$

$$fib.(n + 2) = fib.n + fib.(n + 1).$$

**Speaker:** 48086201

**Problem 7-1**

Derive a program for the following specification.

```

[[
  con N : int {N ≥ 0};
  var r : bool;
  S
  {r ≡ (∃p : p ≥ 0 : N = p3)}
]]

```

**Speaker:** 48086202

**Problem 7-2**

Derive for given  $N$ ,  $N \geq 0$ , a program for the computation of the smallest integer  $x$  that satisfies  $x^3 - 6x^2 + 9x \geq N$ .

**Speaker:** 48086227, 31086830

**Problem 7-3**

Derive a program for the following specification.

```

[[
  con N : int {N ≥ 1}; A, B : array [0..N] of int;
  {A.0 ≤ B.0 ∧ A.N ≥ B.N}
  var r : int;
  S
  {0 ≤ r < N ∧ A.r ≤ B.r ∧ A.(r + 1) ≥ B.(r + 1)}
]]

```

**Speaker:** 31086827, 48086123

**Problem 8-1: The Starting Pit Location Problem**

Given are  $N + 1$  pits located along a circular racetrack. The pits are numbered clockwise from 0 up to and including  $N$ . At pit  $i$ , there are  $p.i$  gallons of petrol available. To race from pit  $i$  to its clockwise neighbor one needs  $q.i$  gallons of petrol. One is asked to design a linear algorithm to determine a pit from which it is possible to race a complete lap, starting with an empty fuel tank. To guarantee the existence of such a starting pit, we assume that

$$(\sum i : 0 \leq i \leq N : p.i) = (\sum i : 0 \leq i \leq N : q.i).$$

**Speaker:** 31086830

**Problem 8-2**

Derive an  $O(N)$  solution to the following problem.

```

[[
  con N : int {N ≥ 0}; X : array [0..N] of int;
  var r : int;
  all equal
  {r = (max p, q : 0 ≤ p ≤ q ≤ N ∧ (∀i, j : p ≤ i ≤ j < q : X.i = X.j) : q - p)}
]]

```

**Speaker:** 48086435