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Deterministic second-order patterns

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Abstract

Second-order patterns, together with second-order matching, enable concise specification of program transformation, and have been implemented in several program transformation systems. However, second-order matching in general is nondeterministic, and the matching algorithm is so expensive that the matching is NP-complete. It is orthodox to impose constraints on the form of higher-order patterns so as to obtain the desirable matches satisfying certain properties such as decidability and finiteness. In the context of unification, Miller's *higher-order patterns* have a single most-general unifier. In this paper, we relax the restriction of his patterns without changing determinism in the context of matching instead of unification. As a consequence, our *deterministic second-order patterns* cover a wide class of useful patterns for program transformation. The time-complexity of our deterministic matching algorithm is linear in the size of a term for a fixed pattern.

Keywords: Second-order pattern matching; Functional programming; Program derivation; Program transformation; Fusion transformation

1. Introduction

Second-order patterns, together with second-order matching, enable concise specification of program transformation, and have been implemented in several program-transformation systems [4,10]. However, second-order matching in general is nondeterministic [9] (there is more than a single match). It is orthodox to restrict the form of higher-order patterns to generate the desirable matches satisfying certain properties such as decidability [12] and finiteness [6].

In the context of unification, Miller defined a certain class of *higher-order patterns* [11] that are deterministic, i.e., patterns have at most a single most-general unifier. He required that free variables should appear as the head of a term where the arguments are distinct bound variables. For example, the pattern $\lambda xy.pyx$ is valid, since the arguments of the free variable p are distinct bound variables y and x. Miller's higher-order patterns, however, are too restrictive for program transformations.

In this paper, we relax the restriction of Miller's patterns by allowing the arguments to be terms, so that our *deterministic second-order patterns* cover a

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wide class of useful patterns for program transformations. Consider, for example, the following fusion transformation rule, which eliminates unnecessary intermediate data structures, in Haskell-like notation [2]:

$$\frac{\forall x, y. x \otimes f y = f(x \oplus y)}{f.\, foldr(\oplus)e = foldr(\otimes)(f e)}$$

which says that a composition of function f with a *foldr* can be fused into a single *foldr*, provided that one can find a function $\otimes$ satisfying the side condition, namely $x \otimes f y = f(x \oplus y)$. The key step of discovering a suitable $\otimes$ is actually a higher-order matching problem. Consider fusing *sum* and $foldr(\lambda x y. x * x : y)[\,]$. To see this, expanding the right-hand side of the fusion condition, we get:

$$\begin{aligned}&\lambda x y. f(x \oplus y)\\ &\quad = \lambda x y.\, sum(x * x : y)\\ &\quad = \lambda x y. x * x + sum\, y.\end{aligned}$$

We then obtain $\otimes$ by matching the resulting term, $\lambda x y. x * x + sum\, y$, with the pattern $\lambda x y. x \otimes sum\, y$. This pattern is beyond Miller's higher-order pattern, and the match $\{\otimes \mapsto \lambda y_1 y_2. y_1 * y_1 + y_2\}$ cannot be obtained by first-order matching. On the other hand, our approach can deal with such patterns and guarantee a unique match.

2. Deterministic second-order patterns

We consider simply-typed lambda *terms*. Terms are built recursively from constants, variables, λ-abstractions, and function applications.

$$T = c \mid v \mid \lambda x.T \mid T T.$$

Given two terms T_1 and T_2, we write $T_1 \trianglelefteq T_2$ if $T_1 =_\alpha T_2$ or T_1 is a proper subterm of T_2, up to α-equivalence. For a term $v T_1 \cdots T_n$, we call v the *head* of the term and $T_1, \ldots, T_n$ the *arguments* of v. A term T is called η-(short) normal if T has no η-redex.

Let *FV* be the function mapping from a term to the set of its free variables. We call the term T *closed* if $FV(T) = \{\}$. For readability we sometimes use infix notation, so $x + y$ denotes the term $(+) x y$.

A substitution (or *match*) is a partial function from variables to closed terms like $\phi = \{p \mapsto \lambda x. x b\}$. The domain of substitution ϕ is written as $dom(\phi)$. Given substitutions ϕ and ψ, the composition of substitutions is written as $\phi \,.\, \psi$. The *quasi-composition* of substitutions $\phi \circ \psi$ is defined as $\phi \,.\, \psi$ if the same variables in domains have the same ranges:

$$\forall v \in dom(\phi) \cap dom(\psi) \,.\, \phi v =_{\alpha\beta\eta} \psi v,$$

where the equality operator $(=_{\alpha\beta\eta})$ is modulo $\alpha\beta\eta$-conversion. Otherwise, $\phi \circ \psi$ is *fail*. We use a special match *fail* that is the zero of match composition, i.e., $fail \circ m = m \circ fail = fail$.

Let T_0 be the set of base types. The set of types T is defined as follows.

$$\alpha \in T_0 \Rightarrow \alpha \in T, \qquad \alpha, \beta \in T \Rightarrow \alpha \to \beta \in T.$$

The *order* of base types T_0 is 1. The order of function types is the maximum of one plus the order of the argument type and the order of the result type. The order of a term is defined as the order of its type.

We are now ready to define our class of patterns, the *deterministic second-order patterns*. As we will see later, matching a pattern in this class with a closed term yields at most one match.

Definition 1 ($\mathcal{DSP}$). A term P is said to be a deterministic second-order pattern ($\mathcal{DSP}$), if the arguments $E_1, \ldots, E_m$ of any free variable occurring in the pattern satisfy the following conditions.

(i) $\forall i. FV(E_i) \neq \{\}$.
(ii) $\forall i, j.\, i \neq j \Rightarrow E_i \ntrianglelefteq E_j$.
(iii) $\forall i.(v \in FV(E_i) \Rightarrow v \notin FV(P))$.
(iv) For all i, E_i is first-order.

The conditions on the arguments are relaxation of Miller's idea from "distinct and bound variables" to "non-mutually embedded terms containing bound variables":

(i) E_i should not be a closed term. For example, the term $p1$ is not a $\mathcal{DSP}$ because the argument 1 is closed.
(ii) For all i, j $(i \neq j)$, E_i is not a subterm of E_j. Therefore, $\lambda x. p x (x + 1)$ is not a $\mathcal{DSP}$ since the argument x is a subterm of another argument $x + 1$.
(iii) Free variables in E_i should be bound in the pattern P. As a result, pq is not $\mathcal{DSP}$.

(iv) For example, $p(\lambda x.x)$ is not $\mathcal{DSP}$ because the argument $(\lambda x.x)$ is more than first-order.

The following are examples of $\mathcal{DSP}$ where c and d are constants.

$\lambda x.p(cx)(dx)$,

$\lambda xy.px(cy)$,

$\lambda x.c(px)(qx)$.

In the rest of the paper, we use the following notational convention. Small letters a, b, c, d represent constants, and other small letters such as p, q, v, x, y, z represent variables. Normally, we use p, q to denote the free variables and x, y, z to denote bound variables. Greek identifiers ϕ, ψ, σ represent matches (substitutions), and capital letters represent terms or patterns. Lists of variables $x_1 \cdots x_l$ are represented by $\bar{x}$, and lists of terms $E_1 \cdots E_m$ by $\overline{E}$. For example, a term $\lambda x_1 \cdots x_l.pE_1 \cdots E_m$ is represented by $\lambda\bar{x}.p\overline{E}$.

3. Deterministic second-order matching

A *pattern* is a term which can contain free variables. Given a pattern P and a closed term T where P and T are $\beta\eta$-normal, a *rule* is a pair of terms written as $P \to T$.

The general *matching problem* is: given a rule $P \to T$, find all the substitutions ϕ such that $\phi P =_{\alpha\beta\eta} T$. Such a substitution ϕ is called a *match*, denoted by $\phi \vdash P \to T$. If there exists at most one match ϕ, we say the match is *deterministic*. If there exists exactly one match, we simply say that the match ϕ is *unique*. If the maximum order of the free variables in P is at most two, we say that matching problem is *second-order*.

Second-order matching is known to be nondeterministic. Algorithms computing all the matches has been proposed in, for example, [9]. The contribution of this paper, on the other hand, is to show that second-order matching is deterministic if we restrict the patterns to $\mathcal{DSP}$.

To begin with, let us introduce the important concept of discharging subterms. Discharging $E_1, \ldots, E_m$ by $y_1, \ldots, y_m$ in T means replacement of all the occurrences of $E_1, \ldots, E_m$ with fresh variables $y_1, \ldots, y_m$ respectively in T. One possible implementation is given in Fig. 1. Intuitively, the function

$$
\begin{array}{l}
\mathit{discharge}\, s\, c = c \\
\mathit{discharge}\, s\, v = \mathit{replace}\, s\, v \\
\mathit{discharge}\, s\, (\lambda x.T_1) = \\
\quad \textbf{let } T' = \mathit{replace}\, s\, (\lambda x.T_1) \\
\quad \textbf{in if } T' = (\lambda x.T_1) \textbf{ then } \lambda x.(\mathit{discharge}\, s\, T_1) \textbf{ else } T' \\
\mathit{discharge}\, s\, (T_1 T_2) = \\
\quad \textbf{let } T' = \mathit{replace}\, s\, (T_1 T_2) \\
\quad \textbf{in if } T' = (T_1 T_2) \\
\qquad \textbf{then } ((\mathit{discharge}\, s\, T_1)(\mathit{discharge}\, s\, T_2)) \\
\qquad \textbf{else } T' \\
\\
\mathit{replace}[\,]\, T = T \\
\mathit{replace}((y, E) : s)\, T = \\
\quad \textbf{if } E = T \textbf{ then } y \textbf{ else } \mathit{replace}\, s\, T
\end{array}
$$

Fig. 1. Discharging algorithm.

$$\mathit{discharge}\big[(y_1, E_1), \ldots, (y_m, E_m)\big]T$$

replaces all the occurrences of $E_1, \ldots, E_m$ with fresh variables $y_1, \ldots, y_m$ respectively in T. That is:

$$
\begin{array}{l}
B = \mathit{discharge}\big[(y_1, E_1), \ldots, (y_m, E_m)\big]T \\
\quad \Rightarrow (\lambda\bar{y}.B)\overline{E} =_{\alpha\beta\eta} T \wedge \forall i. E_i \ntrianglelefteq B.
\end{array}
$$

Lemma 2. *If $P = \lambda\bar{x}.p\overline{E}$ is a $\mathcal{DSP}$ where p is a free variable, then there is at most a single match ϕ such that $\phi \vdash P \to T$.*

Proof. There is no match if T is not transformed into $\lambda\bar{x}.T'$ by $\alpha\eta$-conversion. The match of a rule $p\overline{E} \to T'$ should be in the form $\{p \mapsto \lambda\bar{y}.B\}$. Since free variables in each E_i are bounded in P by Definition 1(iii), by definition of match the equation $(\lambda\bar{y}.B)\overline{E} =_{\alpha\beta\eta} T'$ should be satisfied. Therefore, a term B is a result of replacing $\overline{E}$ with $\bar{y}$ in T'. By Definition 1(i), subterms E_i $(1 \leqslant i \leqslant m)$ contain free variables and if we leave any occurrences of E_i in B, then $\lambda\bar{y}.B$ will contain free variables. This results in generating an illegal substitution containing free variables. Instead, a term B should be obtained by full discharging; replacing all the occurrences of $\overline{E}$ with $\bar{y}$ in T', i.e., $(\lambda\bar{y}.B)\overline{E} =_{\alpha\beta\eta} T' \wedge \forall i. E_i \ntrianglelefteq B$. If some free variables still occur in B after the discharging, this results in illegal substitution. Otherwise, since one argument is not a subterm of another argument by Definition 1(ii), the order of replacing does not affect the result of the match. Thus, the match is obtained deterministically. $\square$

Note that as in the proof, for discharging the arguments of free variables in a $\mathcal{DSP}$, we can use any discharging function satisfying the condition $(\lambda\bar{y}.B)\overline{E} =_{\alpha\beta\eta} T' \wedge \forall i.E_i \ntrianglelefteq B$. In the following, we use the function *discharge* for discharging the arguments from a term. We now give our main theorem below.

Theorem 3. *If P is a $\mathcal{DSP}$, there is at most a single match ϕ such that $\phi \vdash P \rightarrow T$.*

Proof. We use mathematical induction on the structure of the pattern.

Case $(P = \lambda\bar{x}.c\overline{E})$. There is no match if the corresponding term cannot be transformed into $\lambda\bar{x}.c\overline{F}$ by $\alpha\eta$-conversion where the lengths of $\overline{E}$ and $\overline{F}$ are equal. Otherwise, the matching can be decomposed into m matchings $\phi_i \vdash \lambda\bar{x}.E_i \rightarrow \lambda\bar{x}.F_i$ for $i = 1\ldots m$. By the induction hypothesis, each match $\phi_i \vdash \lambda\bar{x}.E_i \rightarrow \lambda\bar{x}.F_i$ is unique or there is no match in which case $\phi_i = \mathit{fail}$. Therefore $\phi' \vdash P \rightarrow T$ is the unique match or there is no match if ϕ' is *fail* where $\phi' = \phi_1 \circ \cdots \circ \phi_m$.

Case $(P = \lambda\bar{x}.v\overline{E} \wedge v \notin FV(P))$. Similar to the first case.

Case $(P = \lambda\bar{x}.v\overline{E} \wedge v \in FV(P))$. By Lemma 2, the match generated by the pattern is unique or there is no match. $\square$

For example, consider $P = \lambda x.p(cx)(dx)$ and the term $T = \lambda x.a(cx)(b(dx))$ where a, b, c and d are constants, p and x are variables, and p occurs free in P. To match P against T, we replace cx and dx with fresh variables y_1 and y_2 in T resulting in the unique match $\{p \mapsto \lambda y_1\, y_2.ay_1(by_2)\}$.

4. An efficient deterministic second-order matching algorithm

Given a rule $P \rightarrow T$ where P is a $\mathcal{DSP}$, the algorithm $\mathcal{M}[\![P \rightarrow T]\!]$, defined in Fig. 2 computes its unique match if it exists. Otherwise it returns the special match fail. For example, $\mathcal{M}[\![c \rightarrow \lambda x.d]\!]$ returns

$$
\begin{aligned}
&\mathcal{M}[\![\lambda x_1 \cdots x_l.P_1 \rightarrow \lambda x_1 \cdots x_o.T_1]\!] \\
&\quad = \mathcal{M}[\![\lambda x_1 \cdots x_l.P_1 \rightarrow \lambda x_1 \cdots x_l.T_1 x_{o+1} \cdots x_l]\!] \\
&\qquad \textbf{if } o < 1 \wedge P_1 \text{ and } T_1 \text{ are not } \lambda\text{-abstraction} \\
&\mathcal{M}[\![\lambda\bar{x}.cE_1 \cdots E_m \rightarrow \lambda\bar{x}.dT_1 \cdots T_m]\!] \\
&\quad = \mathcal{M}[\![\lambda\bar{x}.E_1 \rightarrow \lambda\bar{x}.T_1]\!] \circ \cdots \circ \mathcal{M}[\![\lambda\bar{x}.E_m \rightarrow \lambda\bar{x}.T_m]\!] \\
&\qquad \textbf{if } c = d \\
&\mathcal{M}[\![\lambda\bar{x}.x_i E_1 \cdots E_m \rightarrow \lambda\bar{x}.x_j T_1 \cdots T_m]\!] \\
&\quad = \mathcal{M}[\![\lambda\bar{x}.E_1 \rightarrow \lambda\bar{x}.T_1]\!] \circ \cdots \circ \mathcal{M}[\![\lambda\bar{x}.E_m \rightarrow \lambda\bar{x}.T_m]\!] \\
&\qquad \textbf{if } i = j \\
&\mathcal{M}[\![\lambda\bar{x}.pE_1 \cdots E_m \rightarrow \lambda\bar{x}.T_1]\!] \\
&\quad = \{p \mapsto \lambda y_1 \cdots y_m.B\} \\
&\qquad \textbf{if } \lambda y_1 \cdots y_m.B \text{ is closed} \\
&\qquad \textbf{where} \\
&\qquad y_1, \ldots, y_m \text{ are fresh variables} \\
&\qquad B = \mathit{discharge}[(y_1, E_1), \ldots, (y_m, E_m)]T_1 \\
&\mathcal{M}[\![\_]\!] = \mathit{fail}
\end{aligned}
$$

Fig. 2. The matching algorithm.

fail. In Fig. 2, the first case acts as η-expansion, so, $\mathcal{M}[\![\lambda x.p(cx) \rightarrow c]\!]$ returns $\mathcal{M}[\![\lambda x.p(cx) \rightarrow \lambda x.cx]\!]$. The second and the third cases correspond to the cases in our proof of Theorem 3. If the heads of the pattern and the term are equal and the lengths of their arguments are the same, the rule is decomposed into smaller ones. The fourth case which calls the function *discharge* for exhaustive discharging corresponds to Lemma 2. $\mathcal{M}[\![\lambda ar.a \otimes \mathit{sum}\, r \rightarrow \lambda ar.a * a + \mathit{sum}\, r]\!]$ is an example of the fourth case and computes the following match.

$$
\Big\{\otimes \mapsto \lambda xy.\ \mathit{discharge}\big[(y_1, a), (y_2, \mathit{sum}\, r)\big](a * a + \mathit{sum}\, r)\Big\}.
$$

Formally, we can prove the soundness of the algorithm $\mathcal{M}$, i.e., $\mathcal{M}$ returns the unique match if there exists one.

Theorem 4 (Soundness). *If P is a $\mathcal{DSP}$, then*

$$\phi \vdash P \rightarrow T \Leftrightarrow \phi = \mathcal{M}[\![P \rightarrow T]\!] \wedge \phi \neq \mathit{fail}.$$

Proof. We prove it by induction on the structure of the pattern. The proof is straightforward except for the case where the rule is in the form $\lambda\bar{x}.pE_1\cdots E_m \to \lambda\bar{x}.T_1$. We only show this case.

($\Leftarrow$) Let

$$B = discharge\big[(y_1, E_1), \ldots, (y_m, E_m)\big]T_1.$$

By the property of *discharge*, $(\lambda\bar{y}.B)\overline{E} = T_1$ holds. Therefore, the following matching property holds.

$$\{p \mapsto \lambda\bar{y}.B\} \vdash \lambda\bar{x}.p\overline{E} \to \lambda\bar{x}.T_1.$$

($\Rightarrow$) By Theorem 3, there is at most a single match ϕ such that

$$\phi \vdash \lambda\bar{x}.pE_1\cdots E_m \to \lambda\bar{x}.T_1.$$

The form of the match should be $\phi = \{p \mapsto \lambda y_1 \cdots y_m.B\}$ where

$$\{y_1 \mapsto E_1, \ldots, y_m \mapsto E_m\}B =_{\alpha\beta\eta} T_1.$$

A term B should be made by replacing some E_i with y_i from T_1. By Definition 1(i), E_i contains free variables. Thus if B contains E_i, then ϕ is illegal match. Therefore a term B should be made by replacing all the occurrence E_i with y_i from T_1. This operation matches $B = discharge[(y_1, E_1), \ldots, (y_m, E_m)]T_1$. $\square$

The complexity of our matching algorithm is summarized in the following theorem. Let *size*(t) be a function computing a size of the term t.

$$size\, c = 1,$$
$$size\, v = 1,$$
$$size(t_1t_2) = 1 + size\, t_1 + size\, t_2,$$
$$size(\lambda x.t) = 1 + size\, t.$$

Theorem 5 (Efficiency). *Let P be a $\mathcal{DSP}$, n be the size of the term T, and m be the size of the pattern P. The time complexity of $\mathcal{M}[\![P \to T]\!]$ is $\mathrm{O}(m^2n)$.*

Proof. Except for the second last case of the definition of the matching algorithm $\mathcal{M}$ in Fig. 2, the time complexity of $\mathcal{M}$ is straightforwardly linear in the size of the pattern. For the second last case, the function *discharge* traverses the term, calling the function *replace* that checks for each argument E_i. Since equality check in *replace* needs $\mathrm{O}(m)$, *replace* costs $\mathrm{O}(m^2)$. Therefore *discharge* costs $\mathrm{O}(m^2n)$. $\square$

Since m is often small and bounded, and m is much smaller than n in practice, the algorithm is almost $\mathrm{O}(n)$. For a fixed pattern, the algorithm is $\mathrm{O}(n)$.

5. Conclusion

In this paper, we proposed a class of patterns that have the unique second-order match. We believe that the advantage of determinism is helpful for a user to express his intention to the compiler of program-transformation system in a more precise and predictable way. And it makes possible for the second-order matching to be used in functional languages efficiently [7].

Our pattern is a simple and natural extension of Miller's pattern [11] which has a single most general unifier, and is a sort of a restriction of the two-step valid pattern of Sittampalam and de Moor's [14, 15]. But it is not linear time. They also developed an efficient higher-order matching algorithm, one-step matching algorithm which covers at least complete second-order matches [5,14].

While the second-order matching algorithm is NP-complete [1] and the implementations are expensive [3,9], the restriction on patterns sometimes leads to fast matching algorithms. Second-order pure matching (even unification) with a bounded number of variables is PTIME [16]. Hirata, Yamada and Harao [8] have studied the complexity of various second-order matching. According to their classification, $\mathcal{DSP}$ is a *predicate*, namely any arguments of free variables includes no function variables. The matching problem of a predicate is polynomial if it is *binary function-free*, namely, any function variables are at most 2-ary and it includes no function constants. *Linear context matching*, a restricted form of linear higher-order matching, is $\mathrm{O}(n^3)$ [13]. They solve the problem by dynamic programming with table of size $\mathrm{O}(n^2)$ building from the bottom up. Our restriction makes our matching algorithm fast; given a fixed pattern, the time complexity of our deterministic matching algorithm is linear in the size of a term being matched.

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